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AUTOMATIC CONTROL SYSTEM FOR REPLENISHMENT AT SEA

Gustavo Mario Astorquiza Vivar

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THESIS

AUTOMATIC CONTROL SYSTEM FOR
REPLENISHMENT AT SEA

by

Gustavo Mario Astorquiza Vivar

Thesis Advisor: George J. Thaler

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Concepts of Linear Multivariable Systems Theory are introduced to implement actual modeling techniques. A mathematical model of a one plant -two ship system with multiple inputs and outputs is developed.

Risk of collision during the Approach Phase is the primary consideration.

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A thorough analysis of the ships natural behavior under differing initial conditions is accomplished, thus establishing points of collision.

An Automatic Station Keeping Control System, which guarantees safety and precision during the maneuver is designed and successfully tested.

Automatic Control System
for
Replenishment at Sea.

by

Gustavo Mario Astorquiza Vivar
Lieutenant, Chilean Navy
B.S., Naval Postgraduate School, 1973

Submitted in partial fulfillment of the
requirements for the degree of

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TABLE OF CONTENTS

I. INTRODUCTION 8

II. EQUATIONS OF MOTION 9

- A. MATHEMATICAL MODEL OF A SURFACE SHIP 9
  - 1. Equations of motion for a ship moving in the horizontal plane 11
  - 2. Linearization through Taylor's series expansion 12
  - 3. Nondimensionalization 13
  - 4. Computer simulation 18
  - 5. Controllability and stability of the linear model 21
  - 6. The transfer functions 24
- B. TWO SHIPS UNDER FORCE FIELD EFFECT 26
  - 1. Computer simulation 28
  - 2. Open loop test 31

III. THE AUTOMATIC CONTROL SYSTEM 33

- A. THE MULTIPLE INPUTS, MULTIPLE OUTPUTS PLANT 33
  - 1. Transfer function matrix type number 37
  - 2. Steady state decoupling 41
- B. THE CLOSED LOOP SYSTEM 44
  - 1. Generalization of the multivariable system desired performance 45
  - 2. The controlled plant equations 46

IV. A NEW OPTIMIZATION CRITERIA 48

A. TRIAL AND ERROR PROCEDURE FOR QUASI-OPTIMIZATION OF THE GAIN PARAMETERS 48

B. TRIAL AND ERROR PROCEDURE RESULTS 49

V. CONCLUSIONS 51

FIGURES 52

COMPUTER PROGRAMS 90

LIST OF REFERENCES 120

INITIAL DISTRIBUTION LIST 122

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I• INTRODUCTION

Replenishment at sea, has become a common and necessary practice in almost all the navies of the world.

Underway replenishment as it is understood today, implies the operation of at least two ships steaming at close proximity. Experience and studies over the years have shown that when doing so the ships must withstand hydrodynamic Forces and Moments that tend to alter their dynamics creating situations that involve the risk of collision. Knowing this fact it is of pure logic to think of some automatic device that could prevent and compensate this hydrodynamic phenomena. Unfortunately, the data available today do not assure a complete understanding of it. This limitation does not mean that such automatic system must be seen as impossible; the mathematical model of a ship developed by Abkowitz [1], gives the chance to include these forces and moments as enviromental forcing functions.

Engineer's studies have ranged from the simple case where the dynamic of one ship without external forcing functions is involved to more complex ones where the dynamics of two ships are coupled by the interactive effects.

The most recent one [8], which was used as starting point for this work, treated two ships steaming at close proximity as one system of multivariable inputs and multivariable outputs, and it developed a reliable automatic control system that allowed both ships to keep their relative stations during the replenishment maneuver.

This work, investigates the posibility of improving the above mentioned control system in order to increase its capabilities to the general and more desirable situation where the automatic control system starts operating at the Approach Phase.

II. EQUATIONS OF MOTION

A. MATHEMATICAL MODEL OF A SURFACE SHIP

The derivation of a mathematical model representing the steering and maneuvering of a surface ship in open water is given by Abkowitz [1]. In its general form, the model accounts for non-linear, as well as linear effects.

Bodies moving in a fluid medium are free to move in six degrees of freedom. In order to define the equations of motion, a right hand rectangular coordinate system is established, the origin of which is chosen to be in the body itself, as shown in Figure II-1. The origin and the axis are fixed with respect to the body but movable with respect to another system of coordinates axis fixed in space; it is assumed that at time t=0 (initial time of the problem) both systems coincide.

Straight forward Newtonian Mechanics Laws of motion for a rigid body can be written as two equations -one a force equation and the second a moment equation. The equations are:

$$\text{(external forces)} \quad Fe=\frac{d\,(\text{momentum})}{dt}$$

$$\text{(external moments)} \quad Me=\frac{d\,(\text{angular momentum})}{dt} \qquad \text{(II-1)}$$

The equations describing the ship's six degrees of freedom have been found [1] to be:

$$X=m[\dot{U}-RV+QW-X_G(R^2+Q^2)+Y_G(PQ-\dot{R})+Z_G(PR+\dot{Q})]$$

$$Y=m[\dot{V}-PW+RU+X_G(\dot{R}+PW)-Y_G(P^2+R^2)+Z_G(PQ-\dot{P})]$$

$$Z=m[\dot{W}-QU+PV+X_G(PR-\dot{Q})+Y_G(\dot{P}+QR)-Z_G(Q^2+P^2)]$$

$$L=\dot{P}I_x+(I_z-I_y)QR+m[Y_G(\dot{W}-QU+PV)-Z_G(\dot{V}-PW+RU)]$$

$$M=\dot{Q}I_y+(I_x-I_z)PR+m[Z_G(\dot{U}-RV+QW)-X_G(\dot{W}-QU+PV)]$$

$$N=\dot{R}I_z+(I_y-I_x)PQ+m[X_G(\dot{V}-PW+RU)-Y_G(\dot{U}-RV+QW)]$$

(II-2)

(where the notation $\dot{U}=\partial U/\partial t$ is used)
Satisfying the following equations:

$$\vec{Fe}=\vec{i}X+\vec{j}Y+\vec{k}Z$$

$$\vec{Me}=\vec{i}L+\vec{j}M+\vec{k}N$$

and where the symbols used stand for:

m	Mass of the ship.
X, Y, Z	Components of force in the x, y, z directions.
L, M, N	Components of moment about the x, y, z axis.
U, V, W	Components of velocity in the x, y, z, directions.
X_G, Y_G, Z_G	Distances from the center of gravity to the origin in the x, y, z directions.
P, Q, R	Components of the angular velocity about the x, y, z axis.
I_X, I_Y, I_Z	Moments of inertia about the x, y, z axis.

Equations II-2 describe the reaction of the rigid body to applied forces as a function of the geometric and physical characteristics of the body itself. They do not include any of the applied external forces such as propeller thrust, rudder forces, moments and forces due to the fins (if any), reaction forces of the fluid (hydrodynamic forces), and waves and wind forces.

1. Equations of motion for a ship moving in the horizontal plane

Normally, when dealing with steering and maneuvering of surface ships in open water, the primary motions are considered to take place in the horizontal plane, and vertical motions are neglected. As this is also the case of interest, where only horizontal motions are of concern, the equation in Z, the vertical force equation can be dropped. Under the assumption of calm waters, roll, pitch and heave are all negligibles, i.e.,

$$P=\dot{P}=Q=\dot{Q}=W=\dot{W}=0$$

Hence equations II-2 reduce to

$$X=m[\dot{U}-RV-X_G R^2-Y_G R]$$

$$Y=m[\dot{V}+UR+X_G \dot{R}-Y_G R]$$

$$N=RI_z+m[X_G(\dot{V}+RU)-Y_G(\dot{U}-RV)]$$

(II-3)

and assuming the coordinate axis origin placed at the center of gravity, $X_G=Y_G=0$, equations II-3 become

$X=m[\dot{U}-RV]$, surge

$Y=m[\dot{V}+UR]$, sway

$N=RI_z$, yaw

(II-4)

The left hand sides of equations II-4 represent the forces and moments along and about the coordinate axes, and the right hand sides show the corresponding dynamic reaction.

2. Linearization through Taylor's series expansion

The forces and moments on the left hand side of equations II-1 through II-4 can be expressed as properties of the body, properties of the fluid and motion.

Since steering and maneuvering are of interest, forces and moments are also considered as function of rudder (control surface) deflections and the change in r.p.m of the propeller shaft, furthermore, for a surface ship moving on the horizontal plane no forces or moments are due to orientation changes. Hence:

$$(X,Y,N) \backsim f(u,r,v,\dot{u},\dot{r},\dot{v},\delta,\dot{\delta},\Delta n,\text{etc})$$

(II-5)

In general it is possible to linearize a function f(x) by the use of Taylor's series expansion, thus

$$f(x)=f(x_0)+\frac{\Delta x}{1!}\frac{\partial f(x)}{\partial x}+\cdots\cdots \quad \frac{\Delta x^n}{n!}\frac{\partial^n f(x)}{\partial x^n}$$

where $\Delta x=x-x_0$. For small values of x the second order terms can be neglected and thus considering only the following expression for f(x):

$$f(x)=f(x_0)+\Delta x\,\frac{\partial f(x)}{\partial x}$$

(II-6)

The same principle can be applied for small perturbations in equations II-5, which is a function of many variables. Since the Taylor expansion is written for a particular point, this point is chosen to be an equilibrium position. An equilibrium position is that of straight ahead motion, at constant speed with rudder amidship. The hydrodynamic forces and moments have been found to be:

$$X=X_u\,u+X_v\,v+X_r\,r+X_{\dot{u}}\,\dot{u}+X_{\dot{v}}\,\dot{v}+X_{\dot{r}}\,\dot{r}+X_\delta\delta+X_N\,\Delta n$$

where $\Delta u=u-u_0$ and $\Delta n=n-n_0$ and the subscript 0 is referred to the values of the variables at the initial equilibrium

condition and where all the partial derivatives are evaluated. For Y and N similar expressions hold. Equating the linearized expression for X, Y and N with equations II-4 results in the linearized equations of motion for steering and maneuvering:

$$X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta + X_n \Delta n = m\dot{u}$$

$$Y_u \Delta u + Y_v v + Y_r r + Y_{\dot{u}} \dot{u} + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + Y_\delta \delta + Y_n \Delta n = m(\dot{v} + ru)$$

$$N_u \Delta u + N_v v + N_r r + N_{\dot{u}} \dot{u} + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} + N_\delta \delta + N_n \Delta n = I_z \dot{r}$$

(II-7)

The derivatives X_v, $X_{\dot{v}}$, X_r, $X_{\dot{r}}$, X_δ, Y_u, $Y_{\dot{u}}$, N_u, $N_{\dot{u}}$, vanish for any symmetrical port and starboard shape of ship (symmetry about the xz-plane). This has the effect of decoupling surge from sway and yaw.

Furthermore, considering negligible the effect of Δn in Y and N equations, equations II-7 become:

$$(X_{\dot{u}} - m)\dot{u} + X_u \Delta u = -X_n \Delta n$$

$$(Y_{\dot{v}} - m)\dot{v} + Y_v v + (Y_r - mu) r + Y_{\dot{r}} \dot{r} = -Y_\delta \delta$$

$$(N_{\dot{r}} - I_z)\dot{r} + N_r r + N_{\dot{v}} \dot{v} + N_v v = -N_\delta \delta$$

(II-8)

3. Nondimensionalization

For computer simulation purposes, equations II-8 are used with the nondimensional coefficients of a Mariner ship, whose characteristics are those of Table II-2. The nondimensional nomenclature is shown in Table II-1, and the nondimensional coefficients and conversion factors are shown in Table II-3. [2,3]. Also it is of interest to point out that the digital computer time frame is in nondimensionalized form.

In order to simplify the notation, no special symbols are used for the nondimensional quantities, being understood that only these quantities are of concern.

TABLE II-1

NONDIMENSIONAL NOMENCLATURE

Symbol	Definition
$X'_{\dot{u}}$	Derivative of longitudinal force component with respect to longitudinal acceleration component $\dot{u}$.
X'_u	Derivative of longitudinal force component with respect to longitudinal velocity component u.
Y'_v	Derivative of lateral force component with respect to transverse velocity component v.
$Y'_{\dot{v}}$	Derivative of lateral force component with respect to transverse acceleration component $\dot{v}$.
Y'_r	Derivative of lateral force component with respect to yaw angular velocity component r.
$Y'_{\dot{r}}$	Derivative of lateral force component with respect to yaw angular acceleration component $\dot{r}$.
Y'_δ	Derivative of lateral force component with respect to rudder angle component δ.
N'_v	Derivative of yawing moment component with respect to transverse velocity component v.
$N'_{\dot{v}}$	Derivative of yawing moment component with respect to transverse acceleration component $\dot{v}$.
N'_r	Derivative of yawing moment component with respect to yaw angular velocity component r.
$N'_{\dot{r}}$	Derivative of yawing moment component with respect to yaw angular acceleration component $\dot{r}$.
N'_δ	Derivative of yawing moment component with respect to rudder angle component δ.

X'_n	Derivative of longitudinal force component with respect to change in propeller r.p.m.
Y'_n	Derivative of lateral force component with respect to change in propeller r.p.m.
N'_n	Derivative of yawing moment component with respect to change in propeller r.p.m.
r'	Yawing angular velocity component.
$\dot{r}'$	Yawing angular acceleration component.
U'	Velocity of origin of body axes relative to fluid.
v'	Transverse velocity component of origin of ship axes relative to fluid.
$\dot{v}'$	Transverse acceleration component of ship relative to fluid.
X'	Hydrodynamic longitudinal force (positive direction forward).
Y'	Hydrodynamic lateral force (positive direction to starboard).
m'	Mass of the ship.
u'	Velocity of origin of ship's axes along the x-axis.
$\dot{u}'$	Acceleration of origin of ship's axes along the x-axis.
I'_z	Moment of inertia of the ship with respect to the z-axis.
t'	Time frame.

TABLE II-2

MARINER CHARACTERISTICS AND MAXIMUM VALUES OF SELECTED PARAMETERS

Length, ft	527.8
Beam, ft	76.0
Draft, ft	29.75
Displacement, tons	16800.0
Block coefficient, C_b	0.6
Yaw angle (RAD)	0.26
Yaw velocity (RAD/SEC)	0.0349
Suge velocity (FT/SEC)	5.064
Propeller speed (RPM)	30.0
Rudder deflection (RAD)	0.35

TABLE II-3

NONDIMENSIONAL HYDRODYNAMIC COEFFICIENTS
NUMERICAL VALUES AND CONVERSION FACTORS

Nondimensional Coefficients	Nondimensionalizing Factors	Nondimensional Values*E+05
$(Y'_{\dot{r}}-m'x'_{G})$	$0.5*\rho L^4$	-27
$(X'_{\dot{u}}-m')$	$0.5*\rho L^3$	-850
$X'_{\dot{u}}$	$0.5*\rho L^2 u$	-120
Y'_{v}	$0.5*\rho L^2 u$	-1243
Y'_{δ}	$0.5*\rho L^2 u^2$	270
$(Y'_{\dot{v}}-m')$	$0.5*\rho L^3$	-1500
$(Y'_{r}-m')$	$0.5*\rho L^3 u$	-510
N'_{v}	$0.5*\rho L^3 u$	-351
$N'_{\dot{v}}$	$0.5*\rho L^4$	-19.7
N'_{δ}	$0.5*\rho L^3 u^2$	-126
X'_{n}	$0.5*\rho L^3 u$	4.62
Y'_{n}	$0.5*\rho L^3 u$	-0.52
N'_{n}	$0.5*\rho L^4 u$	0.26
X'_{δ}	$0.5*\rho L^3 u$	0.0
$(N'_{r}-m'x'_{G})$	$0.5*\rho L^4 u$	-227
$(N'_{\dot{r}}-I'_{z})$	$0.5*\rho L^5$	-68
t'	L/u	20.86

Note: ρ= Sea water density (1.9905 slug/ft^3).

4. Computer simulation

If the motion of the ship is to be considered under external perturbations and with acting controls, no further simplifications in equations II-8 are possible.

Thrust, rudder and fin forces and moments are considered control elements, all other forces and moments are not normally controllable inputs, but they must be included in cases where the ship has to be controlled in their presence. To this category belong the interactive forces and moments generated in the case of ships in close underway replenishment stations.

Taking the Laplace Transform of equations II-8 and considering the ship steaming in equilibrium conditions (i.e., steady forward speed $u_0=1$)

$$v(s)[s(Y_{\dot v}-m)+Y_v]+r(s)[sY_{\dot r}+Y_r-m]=-Y_\delta\,\delta(s)$$

$$v(s)[sN_{\dot v}+N_v]+r(s)[s(N_{\dot r}-I_z)+N_r]=-N_\delta\,\delta(s)$$

$$u(s)[s(X_{\dot u}-m)+X_u] \qquad =-X_n\,n(s)$$

(II-9)

Since $r(s)=s\Psi(s)$, equations II-9 become:

$$\frac{v}{s}(s)[s^2(m-Y_{\dot v})-Y_v s]+\Psi(s)[-s^2Y_{\dot r}+s(m-Y_r)]=Y_\delta\,\delta(s)$$

$$\frac{v}{s}(s)[-s^2N_{\dot v}-sN_v]+\Psi(s)[s^2(I_z-N_{\dot r})-sN_r] \qquad =N_\delta\,\delta(s)$$

$$\frac{u}{s}(s)[s^2(m-X_{\dot u})-sX_u] \qquad =X_n\,n(s)$$

(II-10)

letting:

$a^{11}=m-Y_{\dot v}$

$b^{11}=-Y_v$

$c^{11}=0$

$a^{21}=-Y_{\dot{r}}$

$b^{21}=m-Y_{r}$

$c^{21}=0$

$a^{12}=-N_{\dot{v}}$

$b^{12}=-N_{v}$

$c^{12}=0$

$a^{22}=I_{z}-N_{\dot{r}}$

$b^{22}=-N_{r}$

$c^{22}=0$

$a^{33}=m-X_{\dot{u}}$

$b^{33}=-X_{u}$

$c^{33}=0$

Equations II-10 can be written as:

$$\frac{v}{s}(s)[a^{11}s^2+b^{11}s+c^{11}]+\Psi(s)[a^{21}s^2+b^{21}s+c^{21}]=Y_{\delta}\delta(s)$$
$$\frac{v}{s}(s)[a^{12}s^2+b^{12}s+c^{12}]+\Psi(s)[a^{22}s^2+b^{22}s+c^{22}]=N_{\delta}\delta(s)$$
$$\frac{u}{s}(s)[a^{33}s^2+b^{33}s+c^{33}] \qquad =X_{n}n(s)$$

(II-11)

setting:

$v(s)=A(s) \qquad v=\dot{A}$

$\Psi(s)=B(s) \qquad \Psi=B$

$u(s)=C(s) \qquad u=\dot{C}$

$IF1=Y_{\delta}\delta(s)=KA1*D1$

$IF2=N_{\delta}\delta(s)=KB1*D1$

$IF3=X_{n}n(s)=KC1*DN1$

Equations II-11 become:

$$a^{11}\ddot{A}+b^{11}\dot{A}+c^{11}A+a^{21}\ddot{B}+b^{21}\dot{B}+c^{21}B=IF1$$

$$a^{12}\ddot{A}+b^{12}\dot{A}+c^{12}A+a^{22}\ddot{B}+b^{22}\dot{B}+c^{22}B=IF2$$

$$a^{33}\ddot{C}+b^{33}\dot{C}+c^{33}C \qquad =IF3$$

(II-12)

or

$$a^{11}\ddot{A}+a^{21}\ddot{B}=I1$$

$$a^{12}\ddot{A}+a^{22}\ddot{B}=I2$$

$$a^{33}\ddot{C} \qquad =I3$$

(II-13)

where

$$I1=-b^{11}\dot{A}-c^{11}A-b^{21}\dot{B}-c^{21}B+IF1$$

$$I2=-b^{12}\dot{A}-c^{12}A-b^{22}\dot{B}-c^{22}B+IF2$$

$$I3=-b^{33}\dot{C}-c^{33}C+IF3$$

Solution of equations II-13 yields:

$$\ddot{A}=\frac{\begin{vmatrix} I1 & a^{21} & 0 \\ I2 & a^{22} & 0 \\ I3 & 0 & a^{33} \end{vmatrix}}{\Delta} \qquad \ddot{B}=\frac{\begin{vmatrix} a^{11} & I1 & 0 \\ a^{12} & I2 & 0 \\ 0 & I3 & a^{33} \end{vmatrix}}{\Delta} \qquad \ddot{C}=\frac{\begin{vmatrix} a^{11} & a^{21} & I1 \\ a^{12} & a^{22} & I2 \\ 0 & 0 & a^{33} \end{vmatrix}}{\Delta}$$

(II-14)

with

$$\Delta=\begin{vmatrix} a^{11} & a^{21} & 0 \\ a^{12} & a^{22} & 0 \\ 0 & 0 & a^{33} \end{vmatrix}=a^{33}(a^{11}a^{22}-a^{12}a^{21})$$

and replacing the relations between A, B, C and the original variables v,Ψ,u,

$$v=\dot{A}=v_o+\int\ddot{A}dt$$

$$\Psi=B=\Psi_o+\int\dot{B}dt=\Psi+\int[\dot{B}_o+\int\ddot{B}dt]dt$$

$$u=\dot{C}=u_o+\int\ddot{C}dt$$

(II-15)

The transformation from ship to space coordinate system is defined by the following relations, obtained from Figure II-1:

$$\dot{Y}=u\ \mathrm{Sin}\Psi+v\ \mathrm{Cos}\Psi$$
$$\dot{X}=u\ \mathrm{Cos}\Psi-v\ \mathrm{Sin}\Psi$$

(II-16)

from which

$$Y=Y_0+\int\dot{Y}dt$$
$$X=X_0+\int\dot{X}dt$$

(II-17)

Equations II-13 through II-17 were translated into DSL-360 Computer program I. With a constant rudder deflection δ=D1=0.1, the results are shown in Figure II-2 (yaw angle versus time) and Figure II-3 (sway versus surge), the characteristic turning radius of the ship. It must be considered that the described trajectory is valid only for small rudder angles, since linear theory does not give the speed reduction in the turn [1].

5. Controllability and stability of the linear model

These two concepts form a basic background to the subject of the qualitative characteristics in the handling of ships, since each one takes into account the intrinsic properties of a ship and the transfer of initial state of motion to another state in a finite time [10].

a. Controllability [7]

Considering the system

$$\underset{\sim}{x}(t)=\underset{\sim}{A}[\underset{\sim}{x}(t),\underset{\sim}{u}(t),t]\ \text{for}\ t\geq t_0,\ \text{and}\ \underset{\sim}{x}(t_0)=\underset{\sim}{x}_0$$

(II-18)

If there is a finite time $t_1 \geq t_0$ and a control $u(t)$, $t \in [t_0, t_1]$, which transfers the state $\underset{\sim}{x}_0$ to the origin at time t_1, the state $\underset{\sim}{x}_0$ is said to be controllable at time t_0.

If all values of x_0 are controllable for all t_0, the system is completely controllable, or simply controllable.

Kalman [6] has shown that a linear, time invariant system is controllable if and only if the n*mn matrix

$$\underset{\sim}{E}=[\underset{\sim}{B} \,\vdots\, \underset{\sim}{A}\underset{\sim}{B} \,\vdots\, \underset{\sim}{A}\underset{\sim}{B} \text{-------} \vdots\, \underset{\sim}{A}^{n-1}\underset{\sim}{B}]$$

(II-19)

has rank n. If there is only one control input (m=1), a necessary and sufficient condition for controllability is that n*mn matrix $\underset{\sim}{E}$ be nonsingular.

Taking equations II-8 without including the surge equation (because it is decoupled from yaw and sway equations) and dropping out the terms containing the hydrodynamic derivatives Y_r and N_v in the yaw and sway equations [9, Pag. 472] only to apply the Kalman Criterion, the state equations are:

$$\begin{bmatrix} v \\ \\ r \end{bmatrix} = \begin{bmatrix} -b^{11}/a^{11} & -b^{21}/a^{11} \\ \\ -b^{12}/a^{22} & -b^{22}/a^{22} \end{bmatrix} \cdot \begin{bmatrix} v \\ \\ r \end{bmatrix} + \begin{bmatrix} Y \\ \\ N \end{bmatrix}$$

(II-20)

after performing the arithmetic operations

$$\underset{\sim}{E} = \begin{bmatrix} .0027 & -.0018 \\ \\ -.0012 & -.0085 \end{bmatrix}$$

which is rank 2, indicating that complete controllability is assured.

b. Stability

The stability test determines whether or not the ship returns to an established equilibrium condition (straight ahead motion at constant speed), after removing the small disturbance which caused its departure from that equilibrium.

A dynamically unstable ship cannot maintain straight line motion when the rudder is amidship.

The behaviour of the ship can be analized by considering either some introduced disturbance and zero control input ($\delta=0$) or the control acting as a disturbance.

For the first case, and neglecting the surge equation, equations II-8 reduce to

$$v(s)(a^{11}s+b^{11})+r(s)(a^{21}s+b^{21})=0$$

$$v(s)(a^{12}s+b^{12})+r(s)(a^{22}s+b^{22})=0$$

(II-21)

yielding the characteristic equation

$$\begin{vmatrix} a^{11}s+b^{11} & a^{21}s+b^{21} \\ a^{12}s+b^{12} & a^{22}s+b^{22} \end{vmatrix}=0$$

or

$$(a^{11}a^{22}-a^{12}a^{21})s^2+(a^{11}b^{22}+a^{22}b^{11}-a^{12}b^{21}-a^{21}b^{12})s$$
$$+(b^{11}b^{22}-b^{12}b^{21})=0$$

replacing values and rearranging

$$s^2+0.685s+1.016=0$$

(II-22)

both roots belong to the left half s-plane; the ship has fixed control stability with characteristics [11]

$\omega_n = 1.008$

$\varrho = 0.34$

Figure II-4 shows the Root Locus corresponding to equation II-22, which is obtained with Fortran computer program II.

6. The transfer functions

Defining

$K^{11} = a^{11}s^2 + b^{11}s$

$K^{21} = a^{21}s^2 + b^{21}s$

$K^{12} = a^{12}s^2 + b^{12}s$

$K^{22} = a^{22}s^2 + b^{22}s$

$K^{33} = a^{33}s^2 + b^{33}s$

equations II-11 can be written as

$$\begin{aligned} v(s)\,K^{11} + \Psi(s)\,K^{21} &= Y_\delta\,\delta(s) \\ v(s)\,K^{12} + \Psi(s)\,K^{22} &= N_\delta\,\delta(s) \\ u(s)\,K^{33} &= X_n\,n(s) \end{aligned} \qquad \text{(II-23)}$$

solving for $v(s)$, $\Psi(s)$, $u(s)$

$$\frac{v(s)}{s} = \frac{\begin{vmatrix} Y_\delta\,\delta(s) & K^{21} & 0 \\ N_\delta\,\delta(s) & K^{21} & 0 \\ X_n\,n(s) & 0 & K^{33} \end{vmatrix}}{} \qquad \frac{\Psi(s)}{s} = \frac{\begin{vmatrix} K^{11} & Y_\delta\,\delta(s) & 0 \\ K^{12} & N_\delta\,\delta(s) & 0 \\ 0 & X_n\,n(s) & K^{33} \end{vmatrix}}{} \qquad \frac{u(s)}{s} = \frac{\begin{vmatrix} K^{11} & K^{21} & Y_\delta\,\delta(s) \\ K^{12} & K^{22} & N_\delta\,\delta(s) \\ 0 & 0 & X_n\,n(s) \end{vmatrix}}{}$$

(II-24)

where

$$\Delta = \begin{vmatrix} K^{11} & K^{21} & 0 \\ K^{12} & K^{22} & 0 \\ 0 & 0 & K^{33} \end{vmatrix} = K^{33}(K^{11}K^{22} - K^{12}K^{21})$$

and replacing the K's

$$\Delta = s(a^{33}+b^{33})[(s^2(a^{11}a^{22}-a^{12}a^{21})+s(a^{11}b^{22}+a^{22}b^{11}$$
$$-a^{12}b^{12}-a^{21}b^{21})+(b^{11}b^{22}-b^{12}b^{21})]$$

Evaluating the solutions defined by equations II-24

$$\frac{u(s)}{n(s)}=\frac{Ku}{s+t}$$

$$\frac{v(s)}{\delta(s)}=\frac{Kv(s+zv)}{s^2+ps+q}$$

$$\frac{\Psi(s)}{\delta(s)}=\frac{Kr(s+zr)}{s(s^2+ps+q)}$$

(II-25)

where

$$Kv=\frac{Y_\delta a^{22}-N_\delta a^{21}}{a^{11}a^{22}-a^{12}a^{21}}$$

$$Kr=\frac{N_\delta a^{11}-Y_\delta a^{12}}{a^{11}a^{22}-a^{12}a^{21}}$$

$$Ku=\frac{X_n}{a^{33}}$$

$$zv=\frac{Y_\delta b^{22}-N_\delta b^{21}}{Y_\delta a^{22}-N_\delta a^{21}}$$

$$zr=\frac{N_\delta b^{11}-Y_\delta b^{12}}{N_\delta a^{11}-Y_\delta a^{12}}$$

$$t=\frac{b^{33}}{a^{33}}$$

$$p=\frac{a^{12}b^{22}+a^{22}b^{11}-a^{12}b^{21}-a^{21}b^{12}}{a^{11}a^{22}-a^{12}a^{21}}$$

$$q=\frac{b^{11}b^{22}-b^{12}b^{21}}{a^{11}a^{22}-a^{12}a^{21}}$$

(II-26)

The transfer functions defined by equations II-25

and II-26, together with the coordinate transformation given by equations II-16 and II-17 lead to the block diagram representation of the ship, Figure II-5.

The numerical values for the transfer functions are those of table II-4.

TABLE II-4

NUMERICAL VALUES FOR THE TRANSFER FUNCTIONS

Kv= 0.21447
Kr=-1.91507
Ku= 0.00588
zv= 5.76923
zr= 1.29369
p= 0.68467
q= 1.01659
t= 0.14117

B. TWO SHIPS UNDER FORCE FIELD EFFECT

The simple case where the equations of motion of one ship were derived (II-9) did not take in consideration any external field forcing function.

As it has been previously stated, when two ships steam at close proximity, the existence of this Venturi type effect can no longer be ignored [1].

Calvano [4] using available information and digital computer work synthesized the Force and Moment field in a set of curves, which can be considered as valuable information in the designing of an automatic station keeping control system. These curves are shown in Figures II-6 and II-7 respectively.

The major difficulty in handling these curves is the fact that they cannot be written in a simple mathematical form,

but the availability of digital computers minimizes this difficulty to the point where they can be stored as Discrete preknown information (Look-up Table), this is shown in DSL/360 program III, Subroutine Slopes.

All discussion concerning interaction hydrodynamic phenomena, so far, has been for the Mariner which is assumed to be ship A of the two ship system, in Figure II-8.

But hydrodynamic derivatives for both ships, at any relative position of interest are needed to write the equation of motion for the multivariable system. If the two ships are identical the force or moment felt by ship A at a given Dx and Dy are the negatives of the force or moment felt by ship B at the same Dy at Dx=-Dx [4].

Defining

Y=Force in ship A due to ship B

Y=Force in ship B due to ship A

N=Moment in ship A due to ship B

N=Moment in ship B due to ship A

then

$$Y\Big|_{\substack{Dx=\alpha\\ Dy=\beta}} = -Y\Big|_{\substack{Dx=-\alpha\\ Dy=\beta}}$$

and

$$N\Big|_{\substack{Dx=\alpha\\ Dy=\beta}} = -N\Big|_{\substack{Dx=-\alpha\\ Dy=\beta}} \tag{II-27}$$

where α and β represent numerical values in the range of interest of the variables Dx and Dy.

The equations of motion of the two ships, coupled now by the interactive effects can be written as follows:

$$\frac{v_1}{s}(s)\,[a^{11}s^2+b^{11}s+c^{11}]+\Psi_1(s)\,[a^{21}s^2+b^{21}s+c^{21}]=Y_\delta\,\delta_1(s)+Y_1(s)$$

$$\frac{v_1}{s}(s)\,[a^{12}s^2+b^{12}s+c^{12}]+\Psi_1(s)\,[a^{22}s^2+b^{22}s+c^{22}]=N_\delta\,\delta_1(s)+N_1(s)$$

$$\frac{v_2}{s}(s)\,[a^{11}s^2+b^{11}s+c^{11}]+\Psi_2(s)\,[a^{21}s^2+b^{21}s+c^{21}]=Y_\delta\,\delta_2(s)+Y_2(s)$$

$$\frac{v_2}{s}(s)[a^{12}s^2+b^{12}s+c^{12}]+\Psi_2(s)[a^{22}s^2+b^{22}s+c^{22}]=N_\delta\delta_2(s)+N_2(s)$$

$$\frac{u_1}{s}(s)[a^{33}s^2+b^{33}s+c^{33}]=X_n n_1(s)$$

$$\frac{u_2}{s}(s)[a^{33}s^2+b^{33}s+c^{33}]=X_n n_2(s)$$

(II-28)

in which the coupling effect is implicitly defined by equations II-27.

1. Computer simulation

So far, the equations of motion of two ships coupled by the interactive effects (II-28) are in a neat form to be simulated in the digital computer, but it is desirable to impose an additional constraint to reflect a real limitation. There exists a certain time lag between the instant the rudder order is given until the action is completed. This time lag is taken as 0.1 (non dimensionalized time, actual value is 2.08 seconds). Thus the rudder order $\delta_d(s)$ is modified to take into account this time lag.

$$\delta(s)=\frac{1}{str+1}\delta_d(s)$$

(II-29)

where

δ_d is the desired rudder angle
δ is the actual rudder angle
tr is the time lag

a. The equation of motion of ship # 1 (leading ship)
From equations II-28

$$\frac{v_1}{s}(s)[a^{11}s^2+b^{11}s+c^{11}]+\Psi_1(s)[a^{21}s^2+b^{21}s+c^{21}]=Y_\delta\delta_1(s)+Y_1(s)$$

$$\frac{v_1}{s}(s)[a^{12}s^2+b^{12}s+c^{12}]+\Psi_1(s)[a^{22}s^2+b^{22}s+c^{22}]=N_\delta\delta_1(s)+N_1(s)$$

$$\underset{s}{\underline{u_1}}(s)[a^{33}s^2+b^{33}s+c^{33}] \qquad = X_n\, n_1(s)$$

(II-30)

Considering the steering control as it is described by equation II-29

$$(s) = \frac{10\,\delta_{d1}(s)}{s+10}$$

(II-31)

and setting

$$IF11(s) = \frac{Y_\delta \delta_{d1}(s)}{0.1s+1} + Y_1(s)$$

$$IF21(s) = \frac{N_\delta \delta_{d1}(s)}{0.1s+1} + N_1(s)$$

(II-32)

The same procedure followed for the derivation of II-12 through can be applied yielding as before

$$a^{11}\ddot{A}_1 + b^{11}\dot{A}_1 + c^{11}A_1 + a^{21}\ddot{B}_1 + b^{21}\dot{B}_1 + c^{21}B_1 = IF11$$

$$a^{12}\ddot{A}_1 + b^{12}\dot{A}_1 + c^{12}A_1 + a^{22}\ddot{B}_1 + b^{22}\dot{B}_1 + c^{22}B_1 = IF21$$

$$a^{33}\ddot{C}_1 + b^{33}\dot{C}_1 + c^{33}C_1 \qquad = IF31$$

(II-33)

or with

$$I11 = -b^{11}\dot{A}_1 - c^{11}A_1 - b^{21}\dot{B}_1 - c^{22}B_1 + IF11$$

$$I21 = -b^{12}\dot{A}_1 - c^{12}A_1 - b^{22}\dot{B}_1 - c^{22}B_1 + IF21$$

$$I31 = -b^{33}\dot{C}_1 - c^{33}C_1 + IF31$$

(II-34)

equations II-33 can be written as

$$a^{11}\ddot{A}_1 + a^{21}\ddot{B}_1 = I11$$

$$a^{12}\ddot{A}_1 + a^{22}\ddot{B}_1 = I21$$

$$a^{33}C_1 \qquad = I31$$

(II-35)

Solving for $\ddot{A}_1$, $\ddot{B}_1$ and $\ddot{C}_1$

$$\ddot{A}_1 = \frac{a^{22}I11 - a^{21}I21}{a^{11}a^{22} - a^{12}a^{21}}, \quad \ddot{B}_1 = \frac{a^{11}I21 - a^{12}I11}{a^{11}a^{22} - a^{12}a^{21}}, \quad \ddot{C}_1 = \frac{I31}{a^{33}}$$

(II-36)

and the original variables are recovered as before,

$$v_1 = \dot{A}_1 = v_{01} + \int \ddot{A}_1 \, dt$$

$$\Psi_1 = B_1 = \Psi_{01} + \int \ddot{B}_1 \, dt = \Psi_{01} + \int [\, \dot{B}_{01} + \int \ddot{B}_1 \, dt \,] dt$$

$$u_1 = \dot{C}_1 = u_{01} + \int \ddot{C}_1 \, dt$$

(II-37)

so that in the space coordinate system

$$\dot{Y}_1 = u_1 \,\mathrm{Sin}\,\Psi_1 + v_1 \,\mathrm{Cos}\,\Psi_1$$

$$\dot{X}_1 = u_1 \,\mathrm{Cos}\,\Psi_1 - v_1 \,\mathrm{Sin}\,\Psi_1$$

$$Y_1 = Y_{01} + \int \dot{Y}_1 \, dt$$

$$X_1 = X_{01} + \int \dot{X}_1 \, dt$$

(II-38)

b. The equation of motion of ship #2 (trailing ship)

Since, from equations II-28, it is seen that equations for ship #2 are the same as for ship #1 differing only in the subscripts, this derivation is omitted.

Equation of motion of ship #1 and ship #2 were translated into DSL/360 digital computer program III.

A piecewise linear approximation of forces and moments is given by the table look-up and the interpolation Subroutine Slopes. A warning message is printed whenever the distance between the ships become less than 25 feet. For separations greater than 250 feet the ships were considered

to be outside of the range of interest and the forces and moments assumed to be null.

2. Open loop test

As previously mentioned, the existence of risk of collision in the replenishment maneuver is qualitatively well known [3] and it fixes the basis for any attempt of designing an automatic station keeping control system. However a well founded quantitative analysis of this particular subject is almost impossible to find in the current literature.

The open loop test presents a quantitative analysis of the two ship system behaviour, and by choice of different initial conditions it detects the Collision Points. Of course, the simulated situation is ideal and thus only approximates actual physical conditions. The analysis is, however, an excellent approximation to reality.

The simulation is ideal in the sense that human intervention is excluded and only the inherent characteristics of the ships in the hydrodynamic field are evaluated.

Table II-5, indicates the tests performed to the multivariable open loop system using DSL/360 Digital Computer Program III.

TABLE II-5

Test #	U1(0)	U2(0)	DY(0)	DY(0)
1	1.	1.	0.2	-1.
2	1.	1.	0.2	0.
3	1.	1.	0.1	0.
4	1.	1.2	0.2	-1.
5	1.	1.2	0.2	0.

Note that the actual distance is obtained by multiplying the nondimensional length by the length of the ship (i.e., 0.2 <=> 0.2*528 feet).

Figures II-9 through II-20 show the behaviour of the two ships for the conditions pointed out in Table II-5.

Three important conclusions are obtained from the analysis of the Figures.

i) The ideal case, where the equilibrium conditions are set at abeam position at t=0 and released at $t=0^+$ (i.e., $\delta_1=\delta_2=0$) the combined effect of Force and Moment fields tend to pull the ships apart. It is apparent that no collision risk is involved (Test 2, 3, Figures II-12, II-13, II-14, II-15).

ii) The more realistic case where the equilibrium conditions are set at the Approach Phase and then released ($t=0^+$), the combined effect of Force and Moment fields tend to attract both ships and collision is unavoidable. (it must be considered that after $t=0^+$ no human action is involved and what is observed in Test 1 and 4, Figures II-9, II-10, II-11 and Figures II-16, II-17, II-18 correspond to the natural behaviour of the ships).

iii) Test 5, figures II-19 and II-20, shows the Departure Phase, where the speed of ship #2 is allowed to be increased and no risk of collision is observed.

III. THE AUTOMATIC CONTROL SYSTEM

This section starts with the definition of a plant with multiple inputs and multiple outputs and proceeds with mathematical derivations until a model for the automatic control system is obtained.

A. THE MULTIPLE INPUTS, MULTIPLE OUTPUTS PLANT

The initial equilibrium condition for two ships steaming at close proximity indicates that an undetermined δ_0 forcing function is required at t_0 to compensate the effects of $F(\Delta X_0, \Delta Y_0)$ and $M(\Delta X_0, \Delta Y_0)$ which are considered constant. This definition is valid only at $t=0$ since there is not any logic that prevents δ from changing δ_0 to another value that compensates increments of F and M due to variations of ΔX and ΔY. Abkowitz [1] has shown that the force or moment caused by change in any variable is expressed as the product of the derivative of the force with respect to that variable (with all other variables at equilibrium values) and the change in the variable. Using this criteria, at $t=0^+$ it is required to compensate:

$$F(\Delta X,\Delta Y)=\frac{\partial F}{\partial x}\Delta X+\frac{\partial F}{\partial y}\Delta Y$$

and

$$M(\Delta X,\Delta Y)=\frac{\partial M}{\partial x}\Delta X+\frac{\partial M}{\partial y}\Delta Y$$

(III-1)

which represent the "departure" from equilibrium conditions.

Defining

$$\partial F(\Delta X,\Delta Y)=k^{10}\Delta Y+k^{20}\Delta X$$

and

$$\partial M(\Delta X,\Delta Y)=q^{10}\Delta Y+q^{20}\Delta X$$

(III-2)

where k^{10}, k^{20}, q^{10}, q^{20}, represent the rate of change of F and M with respect to ΔX and ΔY measured at ΔX_0 and ΔY_0, the longitudinal and lateral separations from the established equilibrium conditions.

The linear expression for ∂F and ∂M must be defined in terms of the variables u, v, Ψ and then added to the left hand side of equations II-10 under the following assumptions:

a. The two ships are identical.

b. All hydrodynamic coefficients are not affected by the intermingling of the water pressure between the ships, therefore remaining constant.

c. The two ships are considered already as being alongside each other ($\Delta X_0=0$).

d. The forces and moments acting on the ships are equal in magnitude and of opposite sign.

e. The change in forward velocity is considered negligible.

The increments ΔX and ΔY can be expressed as given by equations II-17

$$\Delta X=X_2-X_1=\int(\dot{X}_2-\dot{X}_1)\,dt$$

$$\Delta Y=Y_2-Y_1=\int(\dot{Y}_2-\dot{Y}_1)\,dt$$

Using equations II-16

$$\dot{X}_2-\dot{X}_1=(u_2\,\mathrm{Cos}\Psi_2-v_2\,\mathrm{Sin}\Psi_2)-(u_1\,\mathrm{Cos}\Psi_1-v_1\,\mathrm{Sin}\Psi_1)$$

$$\dot{Y}_2-\dot{Y}_1=(u_2\,\mathrm{Sin}\Psi_2+v_2\,\mathrm{Cos}\Psi_2)-(u_1\,\mathrm{Sin}\Psi_1+v_1\,\mathrm{Cos}\Psi_1)$$

(III-3)

For the small perturbations being considered

$\mathrm{Cos}\Psi_1\approx1$ $\qquad$ $\mathrm{Sin}\Psi_1\approx0$

$$\cos\Psi_2 \approx 1 \qquad \sin\Psi_2 \approx 0$$

and equations III-3 reduce to

$$X_2 - X_1 \approx 0$$

$$Y_2 - Y_1 \approx v_2 - v_1$$

and finally

$$\Delta X \approx 0 \qquad \mathcal{L} \quad \Delta X(s) \approx 0$$

$$\Delta Y \approx \int (v_2 - v_1)\,dt \quad \mathcal{L} \quad \Delta Y(s) = \frac{1}{s}(v_2 - v_1)$$

The equations II-10 are modified to include the interaction effects and become

$$\frac{v_1}{s}(s)\,[a^{11}s^2+b^{11}s+c^{11}]+\Psi_1(s)\,[a^{21}s^2+b^{21}s+c^{21}]+\frac{v_2}{s}(s)\,k=Y_\delta\,\delta_1(s)$$

$$\frac{v_1}{s}(s)\,[a^{12}s^2+b^{12}s+c^{12}]+\Psi_1(s)\,[a^{22}s^2+b^{22}s+c^{22}]+\frac{v_2}{s}(s)\,q=N_\delta\,\delta_1(s)$$

$$\frac{v_2}{s}(s)\,[a^{11}s^2+b^{11}s+c^{11}]+\Psi_2(s)\,[a^{21}s^2+b^{21}s+c^{21}]+\frac{v_1}{s}(s)\,k=Y_\delta\,\delta_2(s)$$

$$\frac{v_2}{s}(s)\,[a^{12}s^2+b^{12}s+c^{12}]+\Psi_2(s)\,[a^{22}s^2+b^{22}s+c^{22}]+\frac{v_1}{s}(s)\,q=N_\delta\,\delta_2(s)$$

(III-4)

where now

$c^{11}=-k$ and $c^{12}=-q$

letting

$$p^{10}=a^{11}s^2+b^{11}s+c^{11}$$

$$p^{20}=a^{21}s^2+b^{21}s+c^{21}$$

$$p^{30}=a^{12}s^2+b^{12}s+c^{12}$$

$$p^{40}=a^{22}s^2+b^{22}s+c^{22}$$

(III-5)

equations III-4 become

$$\frac{v_1}{s}(s)\,p^{10}+\Psi_1(s)\,p^{20}+\frac{v_2}{s}(s)\,k=Y_\delta\,\delta_1(s)$$

$$\frac{v_1}{s}(s)\,p^{30}+\Psi_1(s)\,p^{40}+\frac{v_2}{s}(s)\,q=N_\delta\,\delta_1(s)$$

$$\frac{v_1}{s}(s)\,k+\frac{v_2}{s}(s)\,p^{1}0+\Psi_2(s)\,p^{2}0=Y_\delta\,\delta_2(s)$$

$$\frac{v_1}{s}(s)\,q+\frac{v_2}{s}(s)\,p^{3}0+\Psi_2(s)\,p^{4}0=N_\delta\,\delta_2(s)$$

(III-6)

Equations III-6 show that two ships affected by interaction forces and moments can be described as a multivariable system where the deflection of the rudders δ_1 and δ_2 are the control inputs and the yaw angles Ψ_1 and Ψ_2 the outputs of interest. The next following step has as object to determine the form of the entries of the open loop transfer function matrix $\underset{\sim}{G}$ (whose model is shown in Figure III-1), so that the system can be analyzed and modified, if necessary, to become steady state decoupled -after a transient period of time, a variation introduced in the rudder angle of one ship will not alter the yaw angle of the other .

From figure III-1

$$\underset{\sim}{\delta}(s)=\begin{bmatrix}\delta_1\\ \delta_2\end{bmatrix},\ \underset{\sim}{G}(s)=\begin{bmatrix}g^{11} & g^{22}\\ g^{21} & g^{22}\end{bmatrix}\ \text{and}\ \underset{\sim}{\Psi}(s)=\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix}$$

or

$$\underset{\sim}{\Psi}^{T}(s)=\delta^{T}(s)\,\underset{\sim}{G}(s)$$

(III-7)

Solving for $\Psi^{T}(s)$

$$[\Psi_1\quad \Psi_2]=[\delta_1\quad \delta_2]\begin{bmatrix}g^{11} & g^{12}\\ g^{21} & g^{22}\end{bmatrix}$$

hence

$$\Psi_1=\delta_1 g^{11}+\delta_2 g^{21}$$

$$\Psi_2 = \delta_1 g^{12} + \delta_2 g^{22}$$

(III-8)

and evaluating the gains

$$g^{11} = \frac{\Psi_1}{\delta_1}\bigg|_{\delta_2 = 0} \qquad g^{21} = \frac{\Psi_1}{\delta_1}\bigg|_{\delta_1 = 0}$$

$$g^{12} = \frac{\Psi_2}{\delta_1}\bigg|_{\delta_2 = 0} \qquad g^{22} = \frac{\Psi_2}{\delta_1}\bigg|_{\delta_1 = 0}$$

so $\underset{\sim}{G}(s)$ must be of the type

$$\underset{\sim}{G}(s) = \begin{bmatrix} \dfrac{\Psi_1}{\delta_1} & \dfrac{\Psi_2}{\delta_1} \\ \dfrac{\Psi_1}{\delta_2} & \dfrac{\Psi_2}{\delta_2} \end{bmatrix}$$

(III-9)

1. Transfer function matrix type number

Equation III-9 gave the general form of the Plant matrix $\underset{\sim}{G}$. An explicit expression for the entries of the G matrix is not needed. The steady state decoupling of the states by means of a compensator matrix $\underset{\sim}{Gc}$ requires only the knowledge of the number of pure integrators in the entries of $\underset{\sim}{G}$, this is referred as the type number of the matrix [5].

Solving III-6 for $\Psi_1(s)$ and $\Psi_2(s)$ and recalling III-8

$$\Psi_1(s) = \frac{[g^{11}\ \delta_1(s) + g^{21}\ \delta_2(s)]}{\Delta}$$

(III-10)

$$\Psi_2(s) = \frac{[g^{12}\ \delta_1(s) + g^{22}\ \delta_2(s)]}{\Delta}$$

(III-11)

where

$$\Delta = \begin{vmatrix} p^{10} & p^{20} & k & 0 \\ p^{30} & p^{40} & q & 0 \\ k & 0 & p^{10} & p^{20} \\ q & 0 & p^{30} & p^{40} \end{vmatrix}$$

$$\Delta = (p^{10}p^{40}-p^{20}p^{30})-(qp^{20}-kp^{40}) \tag{III-12}$$

where p^{10} and p^{30} are second order polynomials in s satisfying

$$\lim_{s \to 0} p \neq 0$$

Thus the special case of having either k or q identically zero is avoided.

solving for $\Psi_1(s)$ in III-6

$$\Psi_1(s) = \frac{(N_\delta p^{10}-Y_\delta p^{30})(p^{10}p^{40}-p^{20}p^{30})}{\Delta}\delta_1(s) + \frac{(Y_\delta q-N_\delta q)(kp^{40}-qp^{20})}{\Delta}\delta_1(s) + \frac{(Y_\delta p^{40}-N_\delta p^{20})(kp^{30}-qp^{10})}{\Delta}\delta_2(s) \tag{III-13}$$

where Δ is given by equation III-12.

Replacing p^{10}, p^{20}, p^{30}, p^{40} as indicated by equations III-5 and taking separately $g^{11}(s)$ and $g^{22}(s)$

$$g^{11}(s) = \frac{\Psi_1(s)}{\delta_1(s)} = \frac{N_\delta(a^{11}s^2+b^{11}s+c^{11})(a^{22}s^2+b^{22}s+c^{22})}{\Delta}$$

$$-\frac{N_\delta(a^{11}s^2+b^{11}s+c^{11})(a^{21}s^2+b^{21}s+c^{21})(a^{12}s^2+b^{12}s+c^{12})}{\Delta}$$

$$-\frac{Y_\delta(a^{11}s^2+b^{11}s+c^{11})(a^{12}s^2+b^{12}s+c^{12})(a^{22}s^2+b^{22}s+c^{22})}{\Delta}$$

$$+\frac{Y_\delta(a^{21}s^2+b^{21}s+c^{21})(a^{12}s^2+b^{12}s+c^{12})+Y_\delta qk(a^{22}s^2+b^{22}s+c^{22})}{\Delta}$$

$$-\frac{Y_\delta q^2(a^{21}s^2+b^{21}s+c^{21})+N_\delta k^2(a^{22}s^2+b^{22}s+c^{22})-N_\delta qk(a^{21}s^2+b^{21}s+c^{21})}{\Delta}$$

(III-14)

and from equations II-10, II-11

$c^{21}=c^{22}=0$

then one s can be factored out immediately in the numerator. The independent term must be also checked, if it is equal to zero, then another s can be factored and so on.

Replacing c^{11} by $-k$, and, c^{12} by $-q$, the independent term becomes:

$$N_\delta k^2b^{22}-N_\delta kqb^{21}-Y_\delta kqb^{22}+Y_\delta b^{21}q^2+Y_\delta qkb^{22}$$

$$-Y_\delta q^2b^{21}-N_\delta k^2b^{22}+N_\delta qkb^{21}=0$$

which indicates that a second s can be factored out in the numerator.

Expanding the denominator

$$\Delta=s^2[(as^3+bs^2+cs+d)^2-(es+d)^2]$$

$$\Delta=s^3[a^2s^5+2abs^4+(b^2+2ac)s^3+2(ad+bc)s^2$$

$$+(c^2+2bd-e)s+2d(c-e)]$$

where

$a=a^{11}a^{22}-a^{21}a^{12}$

$b=b^{11}a^{22}+a^{11}b^{22}-a^{21}b^{12}$

$c=b^{11}b^{22}-ka^{22}+a^{21}q-b^{21}b^{12}$

$d=b^{21}q-kb^{22}$

$e=qa^{21}-ka^{22}$

. It can be shown that the independent term never goes to zero for the given values of the hydrodynamic coefficient of the Mariner [9].

Hence, the highest factorable power of s is s^3, and

$$g^{11}(s) = \frac{\Psi_1(s)}{\delta_1(s)}$$

is a type 1 transfer function (i.e., it has one single pole at zero).

Expanding the numerator of $g^{21}(s)$

$$Ng^{21}(s) = s[Y_\delta k(a^{22}s+b^{22})(a^{12}s^2+b^{12}s+c^{12})$$
$$-kN_\delta(a^{21}s+b^{21})(a^{12}s^2+b^{12}s+c^{12})$$
$$+N_\delta q(a^{11}s^2+b^{11}s+c^{11})(a^{21}s+b^{21})]$$

and making the same analysis done before for the independent term:

$$-Y_\delta kqb^{22}+Y_\delta qkb^{22}+kN_\delta b^{21}q-N_\delta qkb^{21}=0$$

which indicates that the highest factorable power of s is s^2. Hence $g^{21}(s)$ is a type 1 transfer function.

The solution of equation III-11 for $\Psi_2(s)$ is

$$\Psi_2(s) = \frac{(N_\delta p^{10}-Y_\delta p^{30})(p^{10}p^{40}-p^{20}p^{30})}{\Delta}\delta_2(s)$$
$$+\frac{(Y_\delta q-N_\delta k)(kp^{40}-qp^{20})}{\Delta}\delta_2(s)$$
$$+\frac{(Y_\delta p^{40}-N_\delta p^{20})(kp^{30}-qp^{10})}{\Delta}\delta_1(s)$$

(III-15)

as could be expected by symmetry, $\Psi_2(s)$ has the same form as $\Psi_1(s)$. As a conclusion, the type number matrix is in general

$$\underset{\sim}{Tp} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(III-16)

2. Steady state decoupling

Once the type number of the plant matrix $\underset{\sim}{G}$ has been found, interest is concentrated in the determination of the cascade compensator matrix.

An important consideration that must be made is the fact that the technique to be used in finding the compensator matrix does not assure by itself stability or good transient response when the feedback path is closed. Figure III-2 shows the closed loop block diagram, where $\underset{\sim}{Gc}$ is the compensator transfer function and $\underset{\sim}{Gp}$ is the plant transfer function, and

$$\underset{\sim}{R}(s) = \begin{bmatrix} \delta_1(s) \\ \\ \delta_2(s) \end{bmatrix}, \quad \underset{\sim}{C}(s) = \begin{bmatrix} \Psi_1(s) \\ \\ \Psi_2(s) \end{bmatrix}$$

Huang and Thaler [5], had shown that assuming that $\underset{\sim}{Gp}$ is stabilized through the configuration of Figure III-2 then the system is steady state decoupled if and only if

$$\lim_{s \to 0} \sum_{i=1, i \neq i}^{n} \frac{r_j}{s^{(kj-1)}} \frac{(\underset{\sim}{I} + \underset{\sim}{Gp}\underset{\sim}{Gc})_{ji}}{\det(\underset{\sim}{I} + \underset{\sim}{Gp}\underset{\sim}{Gc})} = 0$$

for all i=1,---------,n

(III-17)

If $\underset{\sim}{Gc}$ is a diagonal matrix and all inputs are steps (kj=1), the compensator type matrix $\underset{\sim}{Tc}$, which gives the number of integrators required for steady state decoupling of a 2*2 system is obtained by satisfying the conditions:

$N \geq N^{12}, N^{21}$

$N \geq 0$

(III-18)

where

$$M \triangleq Max[(tp^{11}+tc^{11}), (tp^{22}+tc^{22}), (tp^{11}+tp^{22}+tc^{11}+tc^{22}),$$

$$(tp^{12}+tp^{21}+tc^{11}+tc^{22})]$$

$$N^{12} \triangleq tc^{11}+tp^{21}$$

$$N^{21} \triangleq tc^{22}+tp^{12}$$

(III-19)

from equation III-16

$$tp^{11}=tp^{12}=tp^{21}=tp^{22}=1$$

then

$$N^{12}=tc^{11}+1$$

$$N^{21}=tc^{22}+1$$

$$M=Max[(tc^{11}+1), (tc^{22}+1), (2+tc^{11}+tc^{22})]$$

Considering that the plant has four pure integrators, no more are required in the compensator matrix then

$$tc^{11}=tc^{22}=0$$

and

$$N^{12}=N^{21}=1$$

$$M=2$$

satisfying the conditions established by equations III-18

From which:

$$\underset{\sim}{Tc}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \underset{\sim}{Gc}=\begin{bmatrix} g^{11} & 0 \\ 0 & g^{22} \end{bmatrix}$$

(III-20)

By III-16 and with III-20, $\underset{\sim}{G}$ will have the same

type number matrix as $\underset{\sim}{Gp}$, hence

$$\underset{\sim}{T}=\begin{bmatrix} 1 & 1 \\ & \\ 1 & 1 \end{bmatrix}$$

then $\underset{\sim}{G}$ can be written as

$$\underset{\sim}{G}=\begin{bmatrix} \dfrac{P^{11}(s)}{sP_{\triangle}(s)} & \dfrac{P^{12}(s)}{sP_{\triangle}(s)} \\ & \\ \dfrac{P^{21}(s)}{sP_{\triangle}(s)} & \dfrac{P^{22}(s)}{sP_{\triangle}(s)} \end{bmatrix}$$

where $P_{\triangle}(s)$ and $P_{i,j}(s)$, $i,j=1,2$ are polynomials in s such that

$$\lim_{s \to 0} P(s) \neq 0$$

The range of permisible inputs is determined by means of equation III-18

$$\underset{\sim}{I}+\underset{\sim}{G}=\begin{bmatrix} \dfrac{P^{11}(s)+sP_{\triangle}(s)}{sP_{\triangle}(s)} & \dfrac{P^{12}(s)+sP_{\triangle}(s)}{sP_{\triangle}(s)} \\ & \\ \dfrac{P^{21}(s)+sP_{\triangle}(s)}{sP_{\triangle}(s)} & \dfrac{P^{22}(s)+sP_{\triangle}(s)}{sP_{\triangle}(s)} \end{bmatrix}$$

$$\det[\underset{\sim}{I}+\underset{\sim}{G}]=\frac{(P^{11}(s)+sP_{\triangle}(s))\,(P^{22}(s)+sP_{\triangle}(s))}{(sP_{\triangle}(s))}$$

$$-\frac{(P^{21}(s)+P_{\triangle}(s))\,(P^{12}(s)+sP_{\triangle}(s))}{(sP_{\triangle}(s))}$$

$$[\underset{\sim}{I}+\underset{\sim}{G}]_{1,2}=\frac{P^{21}(s)+sP_{\triangle}(s)}{sP_{\triangle}(s)}$$

$$[\underset{\sim}{I}+\underset{\sim}{G}]_{2,1} = \frac{P^{12}(s)+sP\Delta(s)}{sP\Delta(s)}$$

$$\lim_{s\to 0}\left[\frac{1}{s^{(k_i-1)}}\,\frac{[\underset{\sim}{I}+\underset{\sim}{G}]_{1,2}}{\det[\underset{\sim}{I}+\underset{\sim}{G}]}+\frac{1}{s^{(k_i-1)}}\,\frac{[\underset{\sim}{I}+\underset{\sim}{G}]_{2,1}}{\det[\underset{\sim}{I}+\underset{\sim}{G}]}\right]=0$$

and after simplifications

$$\lim_{s\to 0}\frac{s^{-(k_i-2)}P^{21}(s)+s^{-(k_i-2)}P^{12}(s)}{P^{11}(s)P^{22}(s)-P^{21}(s)P^{12}(s)}=0$$

$$(k_1,\ k_2\leq 2)$$

(III-21)

Then the system is steady state decoupled not only for step inputs, but also for ramp (k=2) inputs.

B. THE CLOSED LOOP SYSTEM

Theoretically, the steady state decoupling of the outputs of the plant matrix has been guaranteed and the order of the system kept low because no extra integrators were added in the compensator matrix $\underset{\sim}{Gc}$. However there still remains the problem of satisfying an adequate performance, which in turn is the main purpose of the automatic control system.

Lima [8] solved this problem by modifying the compensator matrix to the form:

$$\underset{\sim}{Gc}=\begin{bmatrix} g^{11}(s+z^{11}) & 0 \\ 0 & g^{22}(s+z^{22}) \end{bmatrix}=\begin{bmatrix} K1+sKT1 & 0 \\ 0 & K2+sKT2 \end{bmatrix}$$

(III-22)

and satisfied the desired performance, which was initially stated in section A.

After an exhaustive analysis of the control system, the idea of increasing its capabilities based on the conclusions

coming next was a feasible and attractive prospect.

i. Existing decoupling in the Surge equations of the multivariable system allowed minor changes in their forward speeds without appreciable effect in their trajectories.

ii. Equations II-27 indicated that no additional difficulties were introduced by changing initial conditions in the X axis (i.e., $\Delta X \neq 0$).

iii. The gains of the compensator matrix obtained by Parameter Optimization Techniques [8] allowed different damping rates in their transient responses.

iv. No automatic speed control was required, since minor changes in speed of the trailing ship were easily made by following the acceleration tables of the ship (i.e., it is assumed that the leading ship forward speed is kept constant).

1. Generalization of the multivariable system desired performance

Only a few words are enough to describe this objective. Reliable work of the automatic control system, capable of keeping the desired relative positions between two ships from the beginning of the replenishment maneuver (Approach Phase) to the end of it (Departure Phase), minimizing the risk of collision. Figure III-3 shows the general case where both ships maneuver.

The variables of interest in the control loop are Ψ_1, Ψ_2, $\dot{\Psi}_1$, $\dot{\Psi}_2$, DY, $\dot{DY}$. Although DX and $\dot{DX}$ are also variables that must be controlled, they do not affect in the control loop, their action is controlled manually by changing the forward speed of ship #2 until DX is reduced to zero.

The design procedure is an extension of the work presented by Lima [8], in which the ships were initially considered steaming alongside at a lateral separation greater than that required to perform the replenishment

maneuver. The optimal gains as given in Table III-1 were obtained as result of choosing the desired trajectories for each ship and defining a cost function to be minimized. These values were kept the same for these studies, however, the initial conditions, specifically U20, DX, and Y1 and Y2 were changed in order to include the Approach Phase.

2. The controlled plant equations

The block diagram of the controlled plant is shown in figure III-4, and the following relations are obtained:

$$\delta_{d1}(s) = \frac{1}{trs+1}\left[(KTY1s+KY1)\, Y1(s) + (KT1s+K1)\, \Psi_1(s) \right]$$

(III-23)

$$\delta_{d2}(s) = \frac{1}{trs+1}\left[KY2(DY-DFIN) + KTY2sDY + (KT2s+K2)\, \Psi_2(s) \right]$$

(III-24)

where DFIN is the Desired Final Lateral Separation.

The fact that there exists limitations in the maximum values of the rudder angles [3] imposed an additional restriction, which is

$|\delta| \leq 0.349$ or $(\leq 20^\circ)$; for $|\delta| \geq 0.349$

II-23 and II-24 become

$$\delta_{d1}(s) = \frac{1}{trs+1} * (0.349)$$

(III-25)

$$\delta_{d2}(s) = \frac{1}{trs+1} * (0.349)$$

(III-26)

Digital computer program #4, which is program #3 modified by including equations III-23 through III-25 allows us to make the simulation of the multivariable system now implemented by the station keeping automatic control system.

TABLE III-1

FEEDBACK LOOP GAINS

Parameter	Run #1	Run #2
U10	1.	1.
U20	1.2	1.2
DY(0)	0.4	0.36
DX(0)	-1.	-1.
DFIN	0.24	0.3
K1	2.975	3.069
KT1	2.775	3.080
K2	2.917	2.853
KT2	2.042	2.141
KY1	3.008	3.320
KTY1	2.783	2.555
KY2	1.511	1.533
KTY2	2.878	2.608

Figures III-5 through III-10 show the computer simulation results of the controlled plant. Both ships reached the desired steady trajectories allowing a safe, as well as precise, maneuver, although a residual lateral separation error exists (0.02 <=> approximately 11 feet), which can be easily removed by indexing in the hardware. Note that these are not solely I. C. curves but are tests of system behaviour as ship 2 approaches ship 1 at a forward speed 20% greater than that of ship 1. The initial large excursion of ship 1 is due in part to the large moment caused by the pressure field, and in part by the chosen initial conditions.

IV. A NEW OPTIMIZATION CRITERIA

The preceeding section led to the final design of the automatic control system. A basic assumption was made in the sense that the feedback gains were kept unchanged from the optimals obtained by Lima [8].

Computer simulation demonstrated that the assumption made was good enough to allow the two ship system to satisfy the desired performance. However the trajectories can no longer be said to be optimal, since completely different initial conditions were imposed. To overcome this fact two techniques were considered -the first one, to reformulate the optimization study incorporating the new initial conditions, and, the second, to use the actual gains and to originate a systematic procedure, basically Trial and Error, to determine not optimal, but quasi-optimal gains.

Because it was expected that the range of variation of the parameters would be reasonably small, the second procedure was adopted. Of course, the Trial and Error procedure has the disadvantage that many statistical combinations are ignored, but, on the other hand experience gives a high degree of confidence in a systematic computer trial procedure.

A. TRIAL AND ERROR PROCEDURE FOR QUASI-OPTIMIZATION OF THE GAIN PARAMETERS

Selecting the time basis trajectories shown by Figures III-8 and III-9 the new objective is to smooth Sway and Yaw transient responses. Note that the same procedure applies to Figures III-5 and III-6.

From Figure III-4, a Procedure Flow Chart was made which includes all the possibilities of discrete changes in the K's parameters in the range 0 to 100% by increasing its values.

The possibility of decreasing the K's values was left out because it was estimated, based on linear theory, that it would not meet the requirements.

Figure IV-1 shows the Trial Procedure Flow Chart.

From computer simulation, many runs were done using DSL/360 computer program #5 but after the selection of the "best set", graphs of Run #1 through Run #4 were considered important enough to be included for purpose of analysis. Note that all of these simulation runs were made with the trailing ship approaching the leading ship at a speed 20% greater than that of the leading ship.

B. TRIAL AND ERROR PROCEDURE RESULTS

Table IV-1 shows the gain parameters for the four main runs of the Trial Procedure Flow Chart. Figures IV-2 through IV-9 are the computer output for this stepped procedure.

The decision of the best set of K's was somewhat subjective and based principally on the shape of the corresponding computer output curves. A qualitative analysis of the Sway and Yaw curves, however, indicates that there is a definitely improvement in the smoothness of the transient, Figures IV-8 and IV-9 being considered the best set, as may be seen by comparison with Figures III-8, III-9.

Figures IV-8 and IV-9 are the most important because they assure that the systematic Trial and Error procedure was an excellent tool to obtain quasi-optimal gains for the specific initial conditions of the Generalized Desired Performance described in section III.

TABLE IV-1

FEEDBACK LOOP GAINS FROM TRIAL PROCEDURE

Parameter	Run #1 KT1+50%	Run #2 KTY1+50%	Run #3 KY2+25%	Run #4 K1+15%
U10	1.	1.	1.	1.
U20	1.2	1.2	1.2	1.2
DY(0)	0.36	0.36	0.36	0.36
DX(0)	-1.	-1.	-1.	-1.
DFIN	0.3	0.3	0.3	0.3
K1	3.069	3.069	3.069	3.52935
KT1	4.62	4.62	4.62	4.62
K2	2.853	2.853	2.853	2.853
KT2	2.141	2.141	2.141	2.141
KY1	3.320	3.320	3.320	3.320
KTY1	2.555	3.8325	3.8325	3.8325
KY2	1.533	1.533	1.91625	1.91625
KTY2	2.608	2.608	2.608	2.608

V. CONCLUSIONS

It has been demonstrated that the intrinsic dynamics of ships steaming at close proximity is unstable, the Approach Phase of the replenishment maneuver being of particular interest, where, excluding human intervention, collision is unavoidable.

The mathematical modeling of two ships as one system with multi-inputs and multi-outputs has proven to be a powerful technique in the study of the dynamic behaviour of vehicles coupled by a common media.

The station keeping control system, initially based on a previous concept, was tested under different conditions, and, significant modifications were introduced, mainly in the gain parameters of the feedback loops, to satisfy the aims of safety as well as precision in the whole maneuver.

To avoid increasing the order of the general system equations, no forward speed control was included, being assumed that the forward speed would be controlled manually as is actually done.

Even though the design of the hardware was not intended, it is estimated that a Modular System composed of a Doppler Sonar, a Gyroscopic Element and probably a Digital Logic would be suitable in its realization.

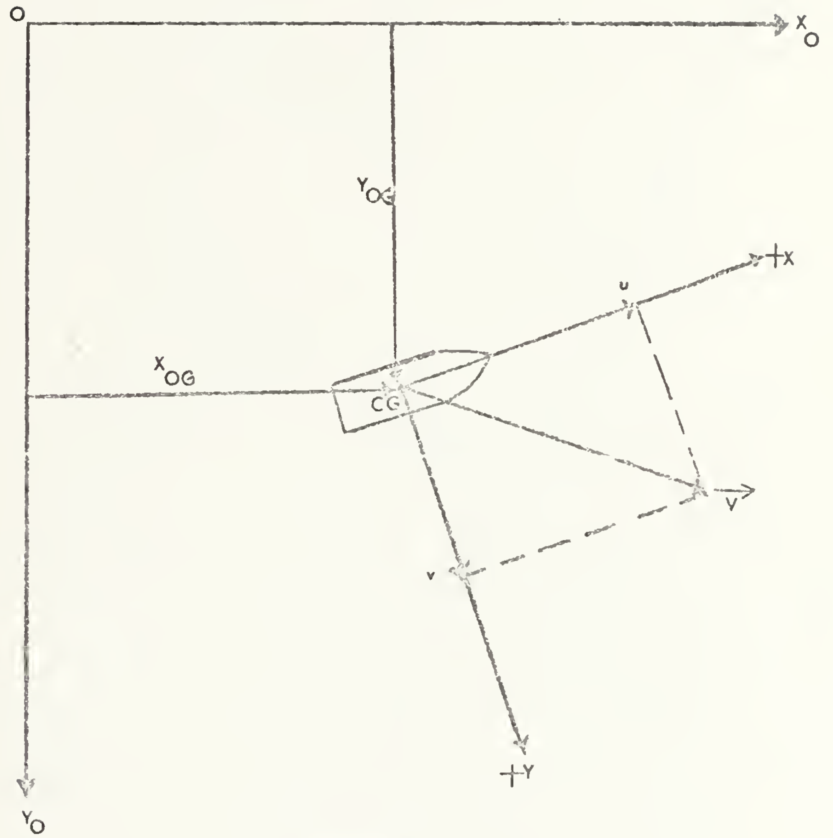


Figure II-1. Orientation of the Space Axis (Xo, Yo) and the Moving Axis (X, Y).

FIGURE II-2. YAW ANGLE VS. TIME.

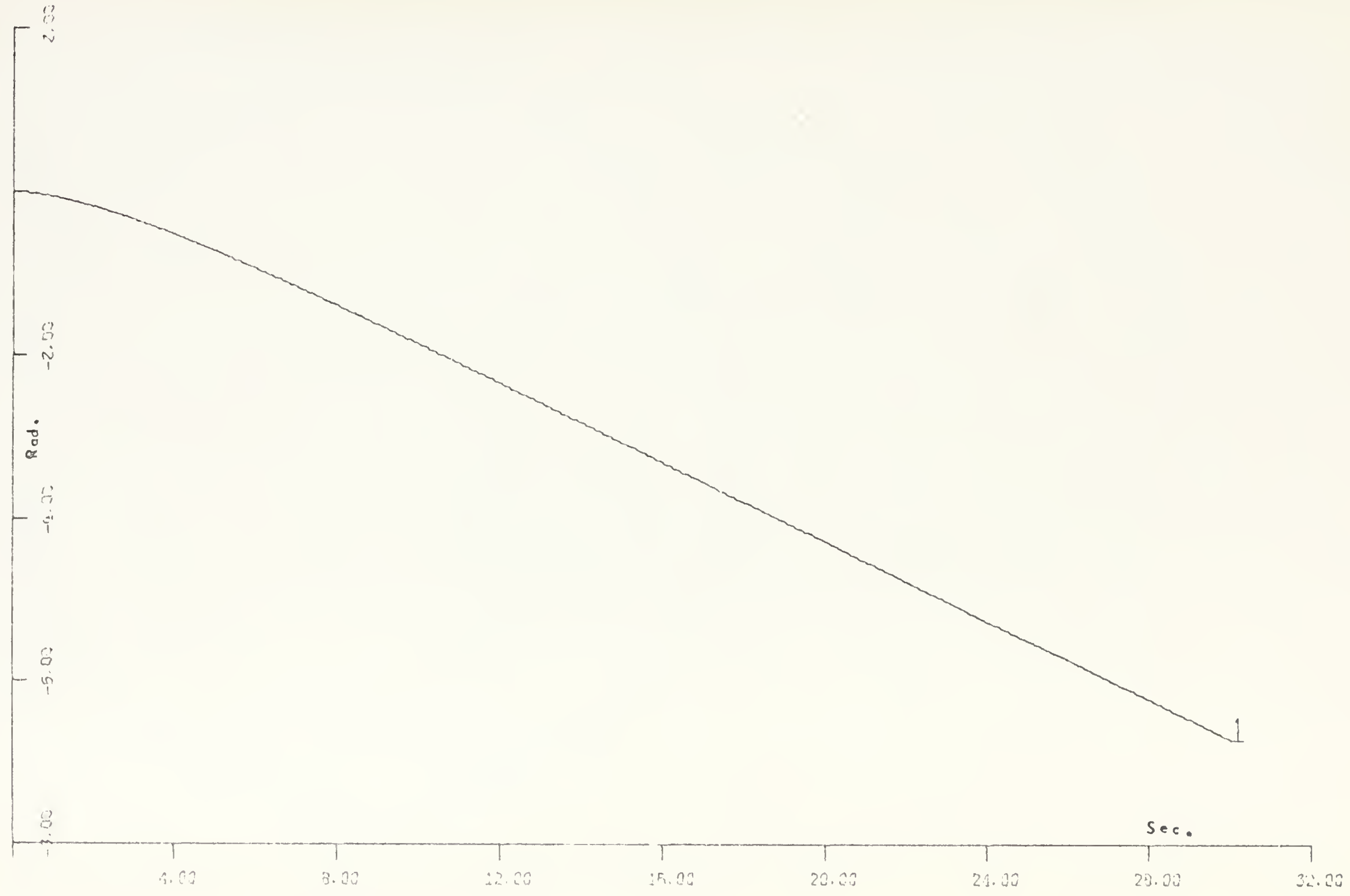
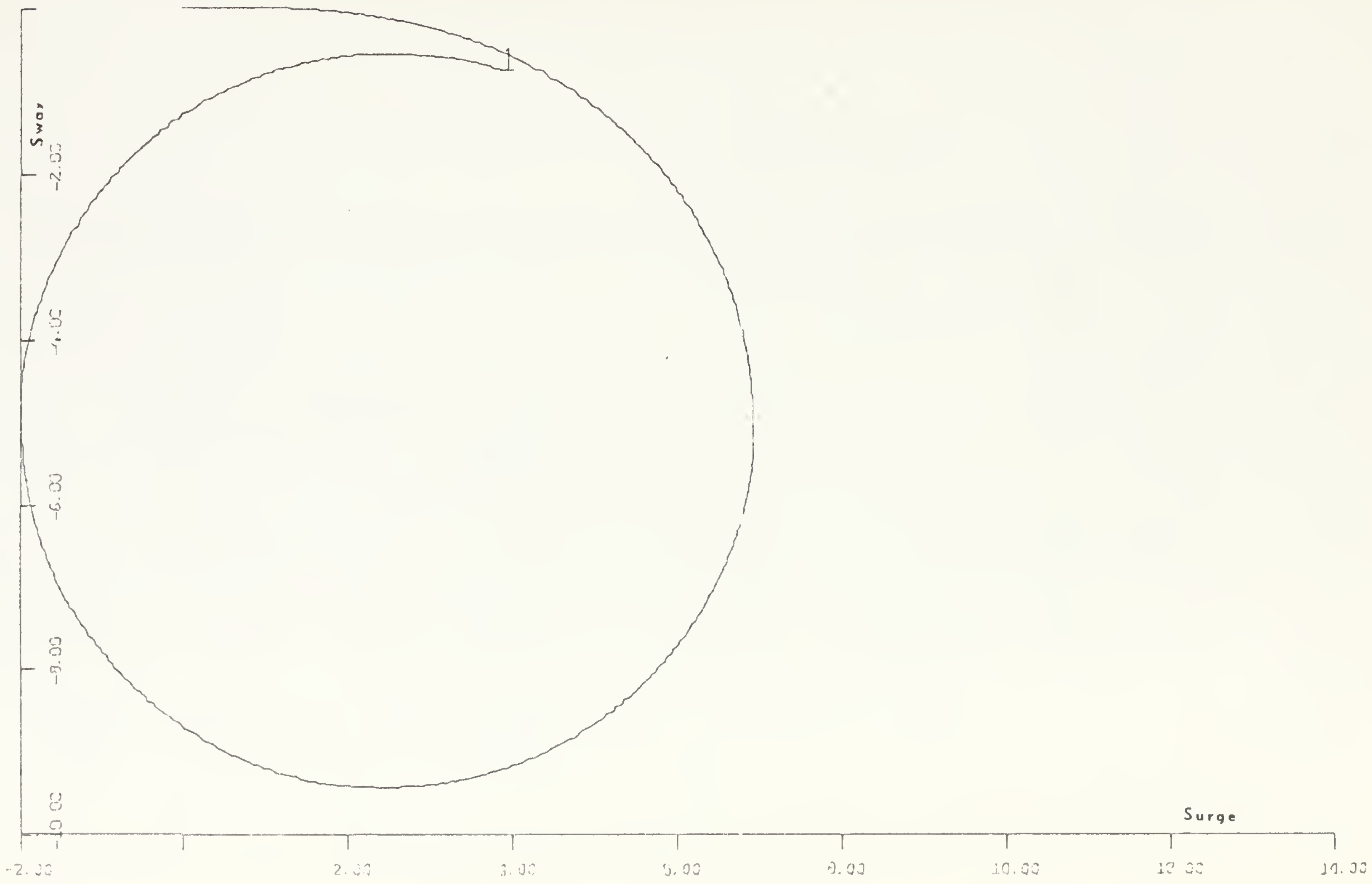
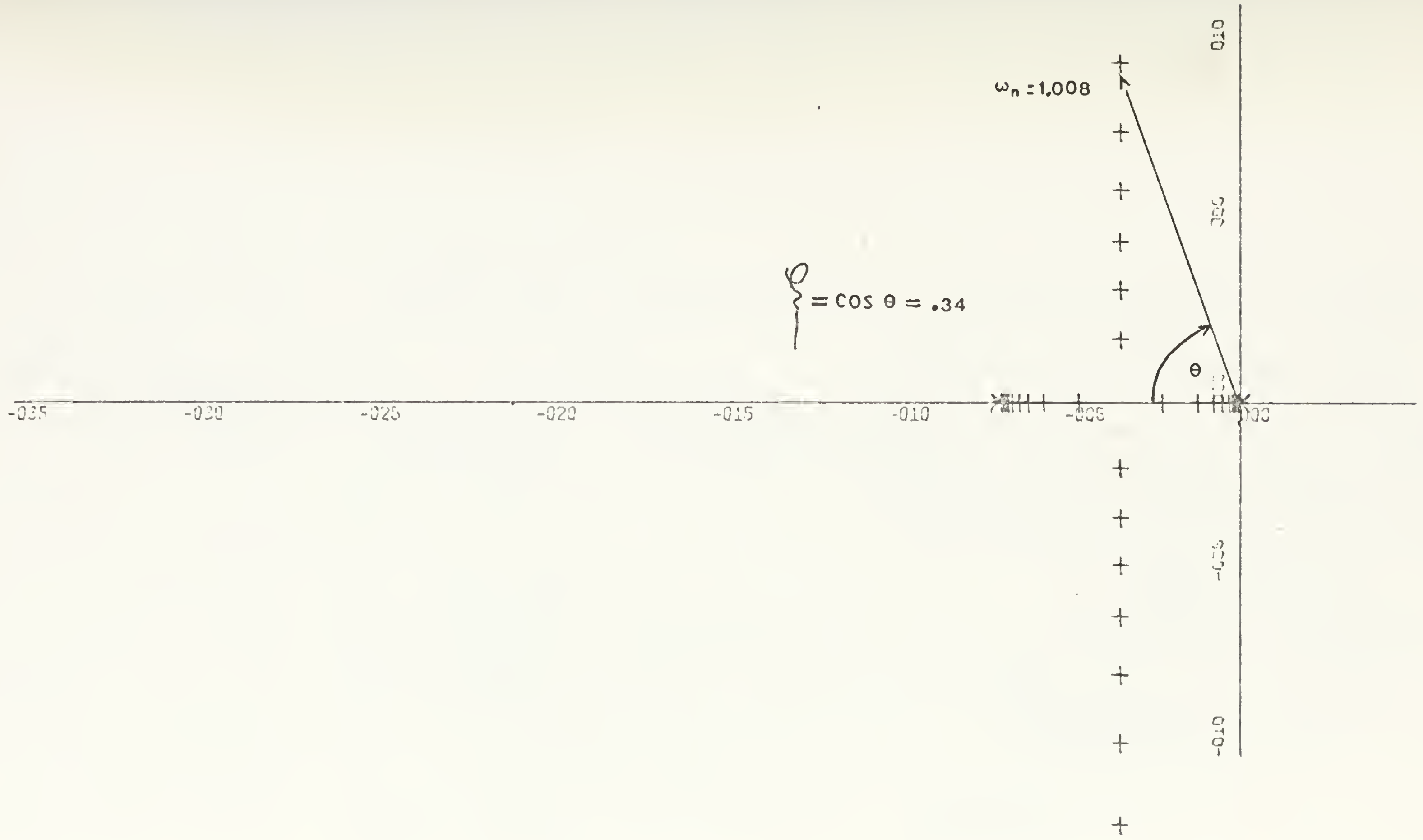


FIGURE II-3. CHARACTERISTIC TURNING RADIUS.





X-SCALE=5.00E-01 UNITS INCH.
Y-SCALE=5.00E-01 UNITS INCH.

FIGURE II-4 .
STABILITY CHARACTERISTICS .

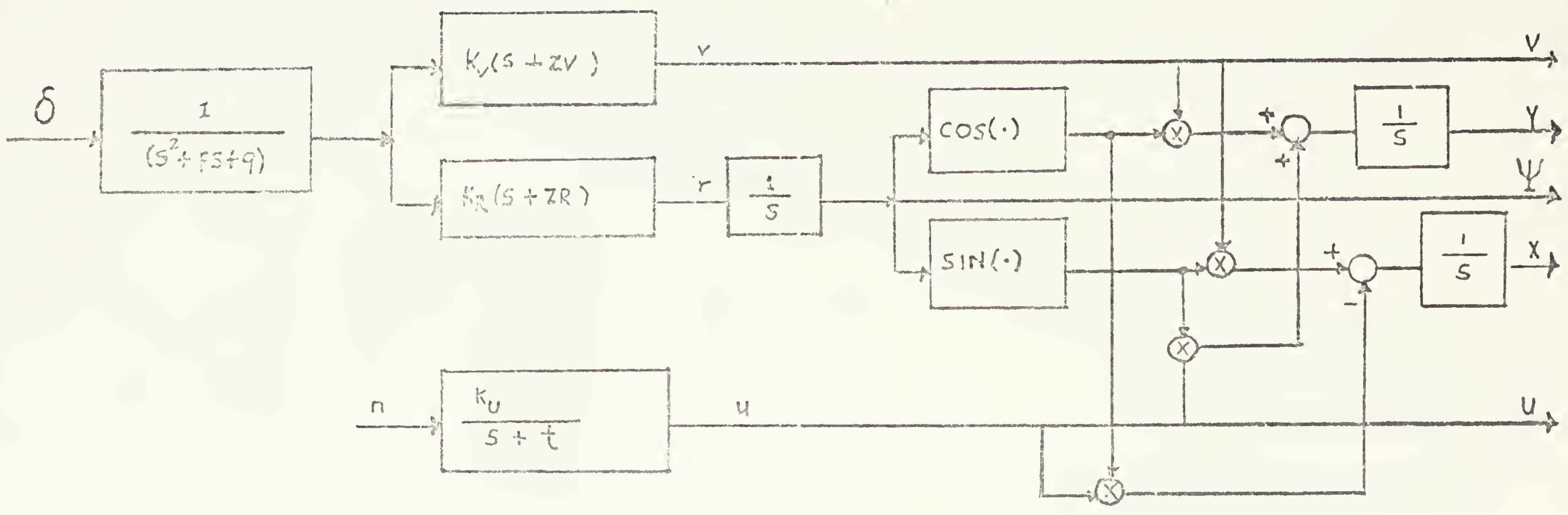


Figure II-5. The One Ship Open Loop Block Diagram.

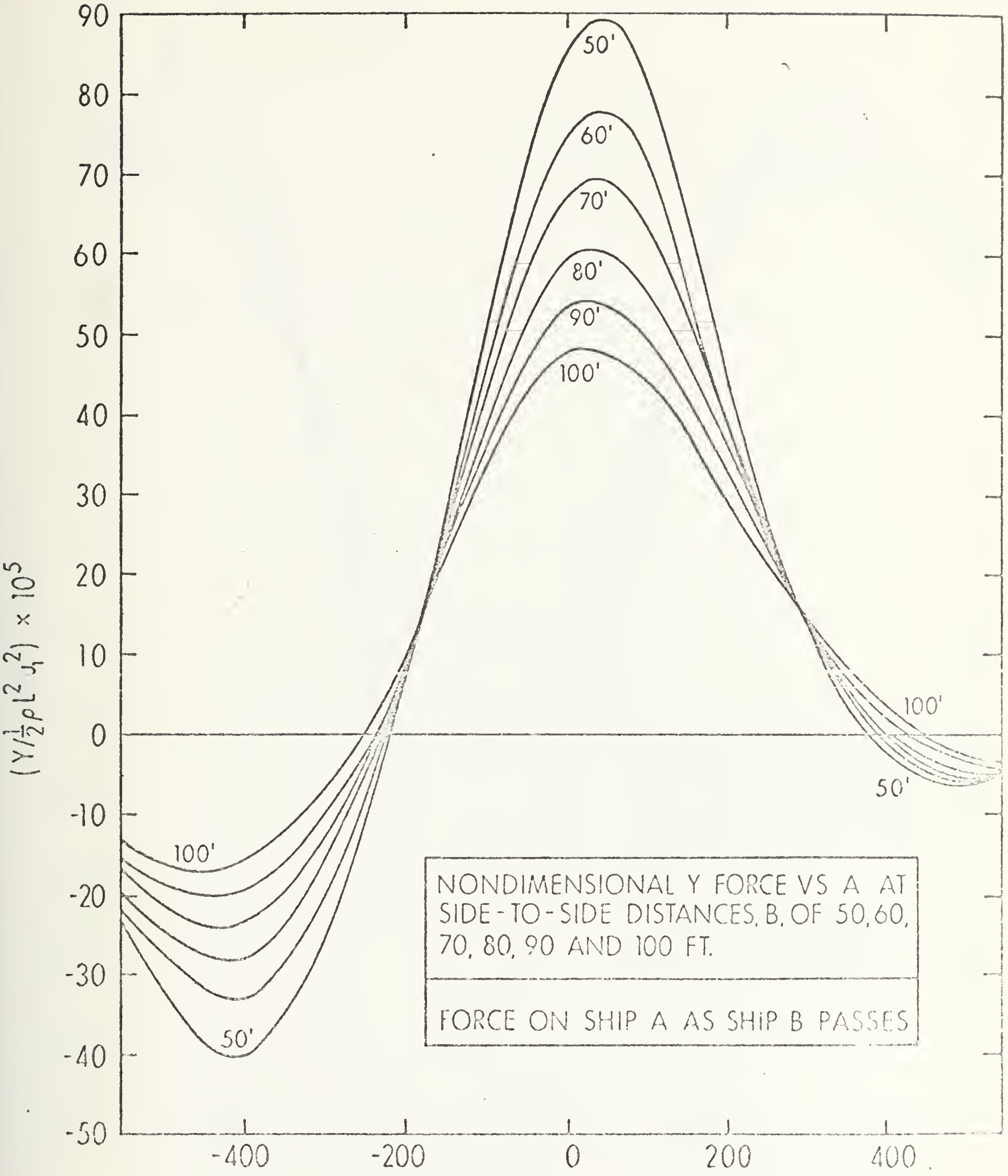

90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
(Y/½ρL² J₁²) × 10⁵
-400
-200
0
200
400
50'
60'
70'
80'
90'
100'
100'
50'
100'
50'
NONDIMENSIONAL Y FORCE VS A AT SIDE-TO-SIDE DISTANCES, B, OF 50, 60, 70, 80, 90 AND 100 FT.
FORCE ON SHIP A AS SHIP B PASSES

FIGURE II-6

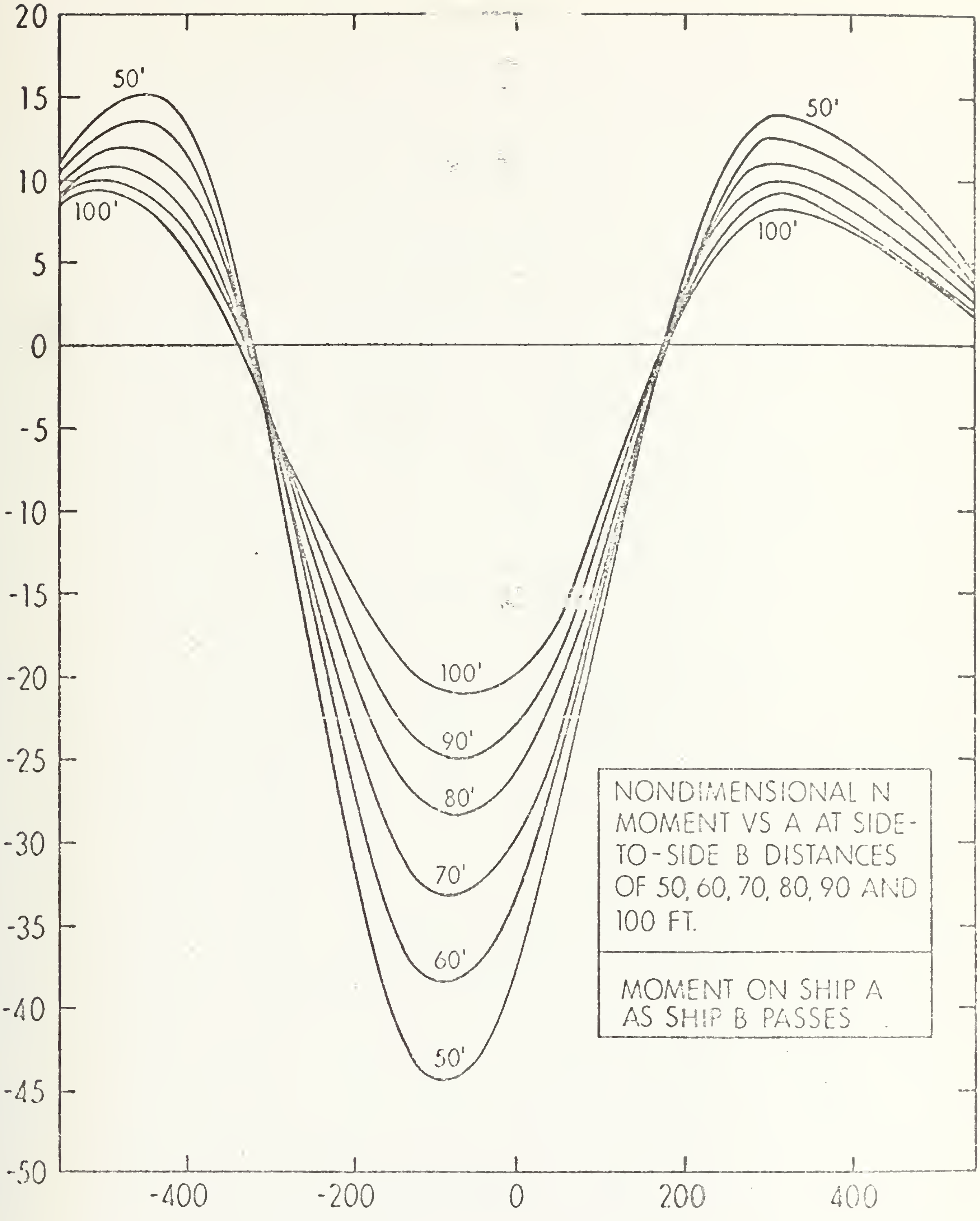

20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
$(N/\frac{1}{2}\rho L^3 u_1^2) \times 10^5$
-400
-200
0
200
400
50'
100'
50'
100'
100'
90'
80'
70'
60'
50'
NONDIMENSIONAL N MOMENT VS A AT SIDE-TO-SIDE B DISTANCES OF 50, 60, 70, 80, 90 AND 100 FT.
MOMENT ON SHIP A AS SHIP B PASSES

FIGURE II-7

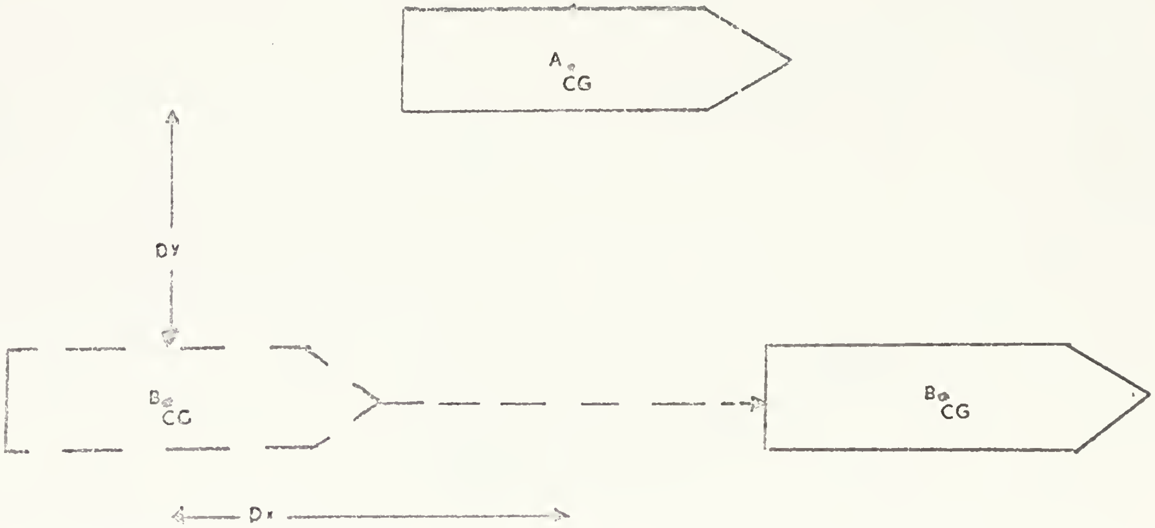


Figure II-8 The two ship system

FIGURE II-9. SWAY VS. TIME. TEST #1.

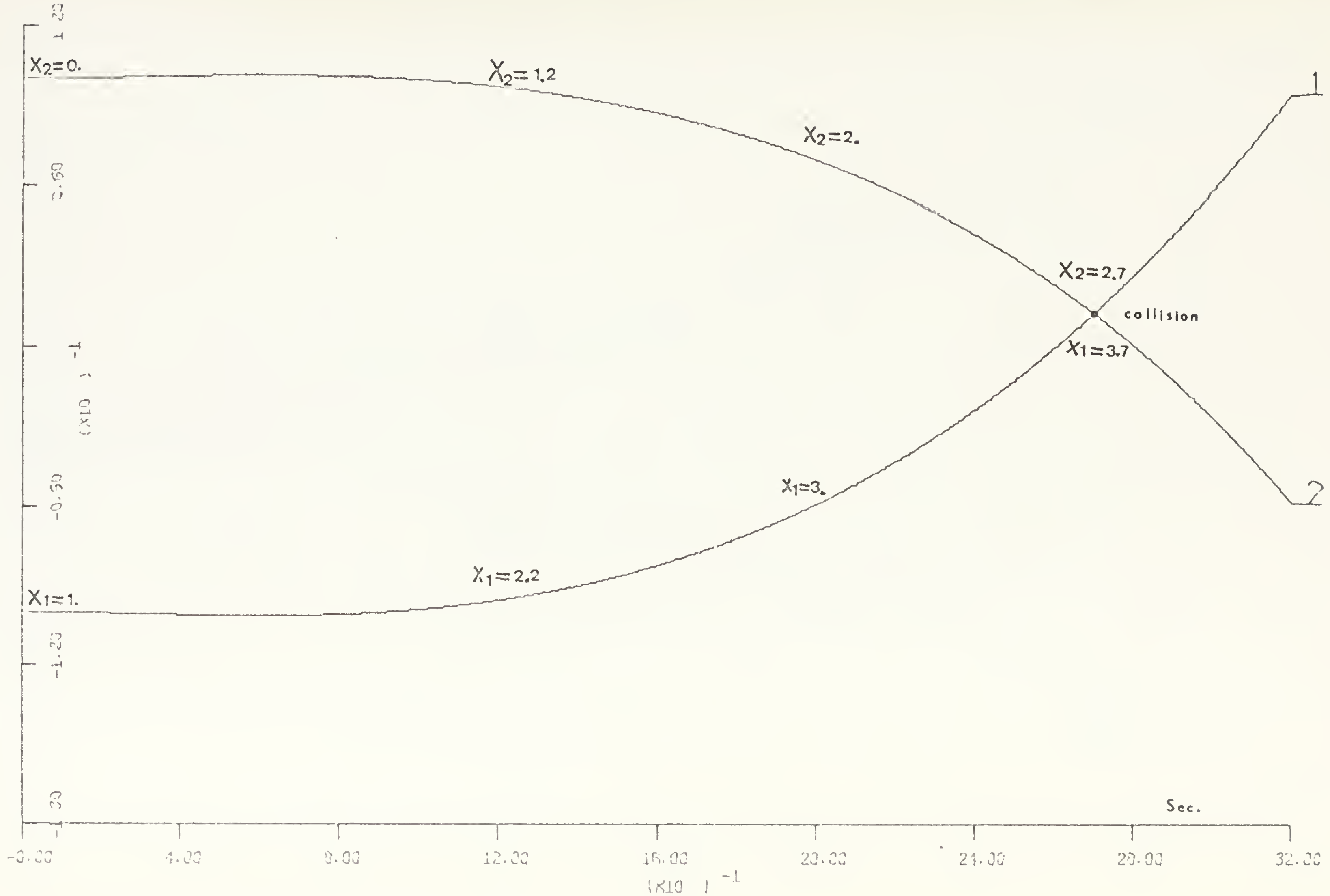
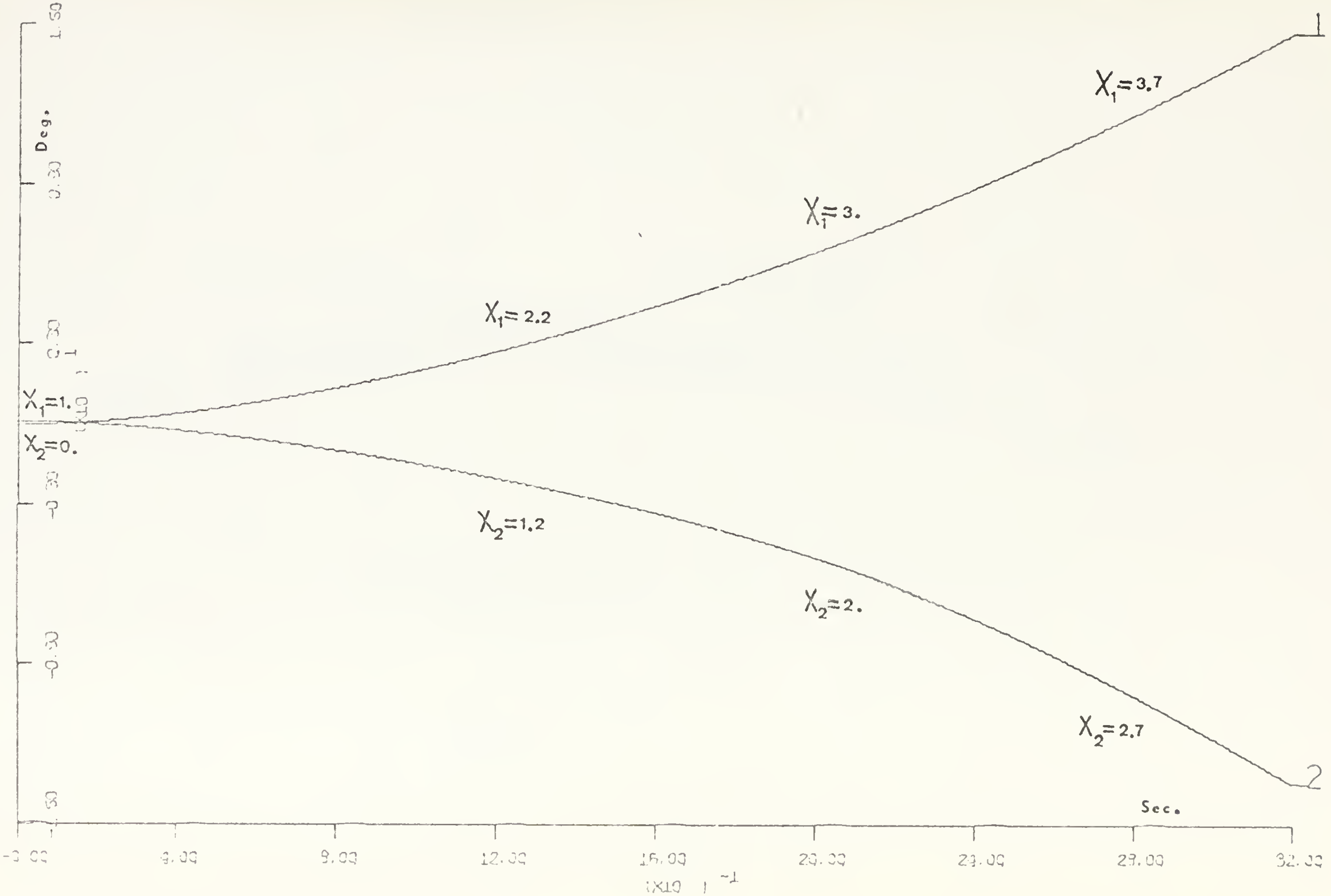


FIGURE II-10. YAW VS. TIME. TEST #1.



Test1 - Collision

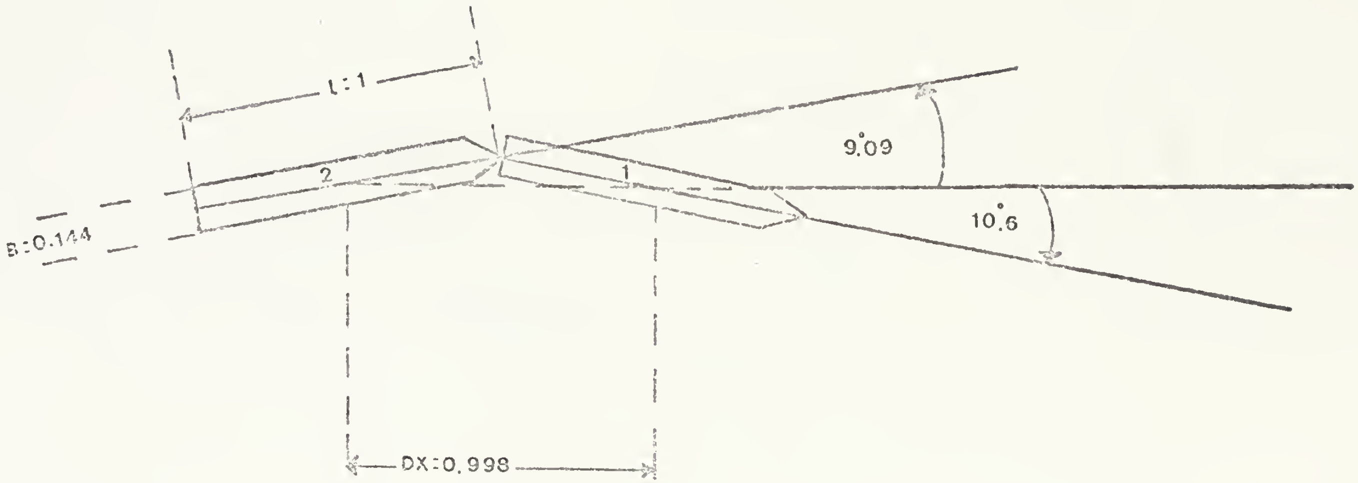


Figure II-11

FIGURE II-12. SWAY VS. TIME. TEST #2.

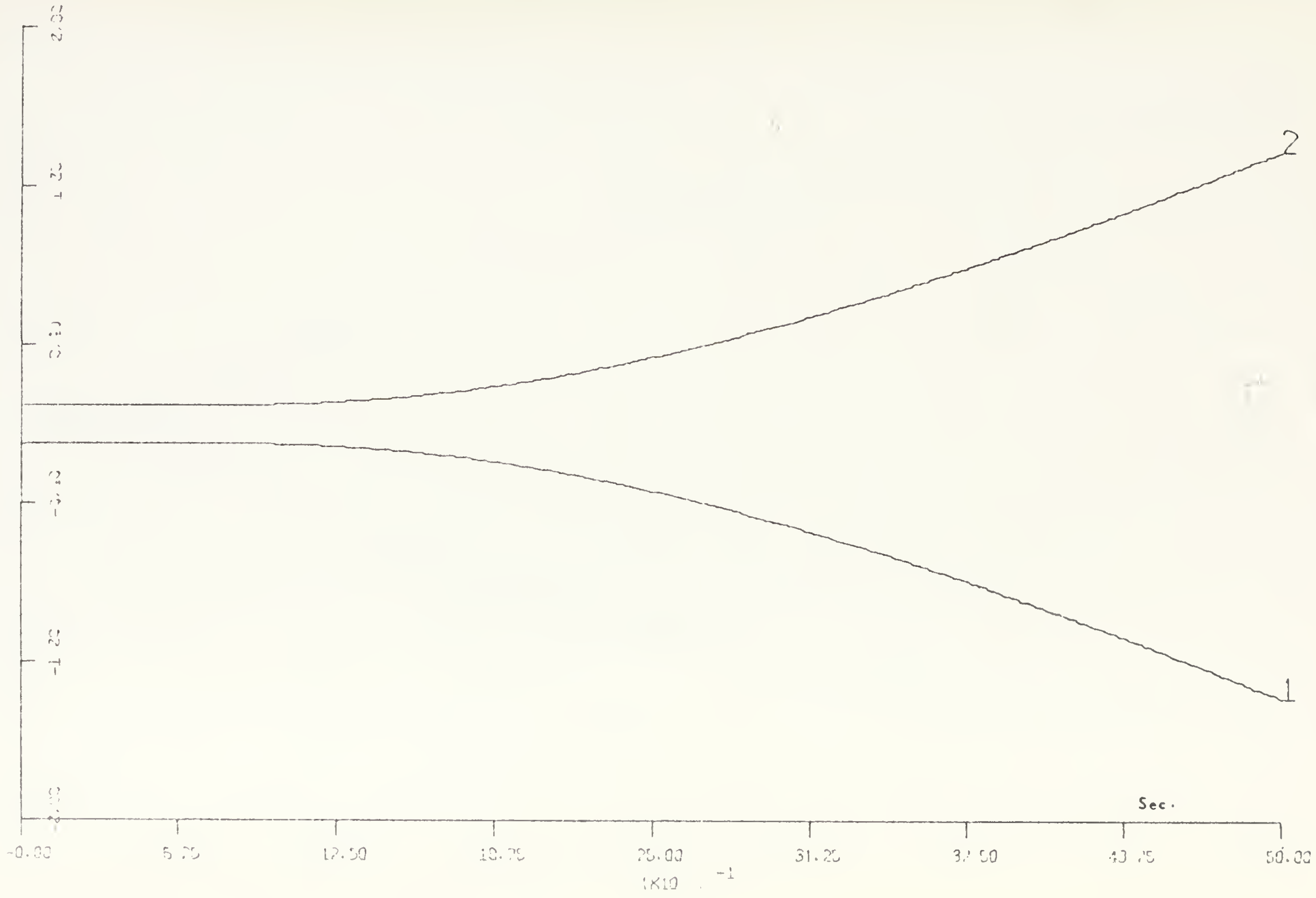


FIGURE II-13. YAW VS. TIME. TEST #2.

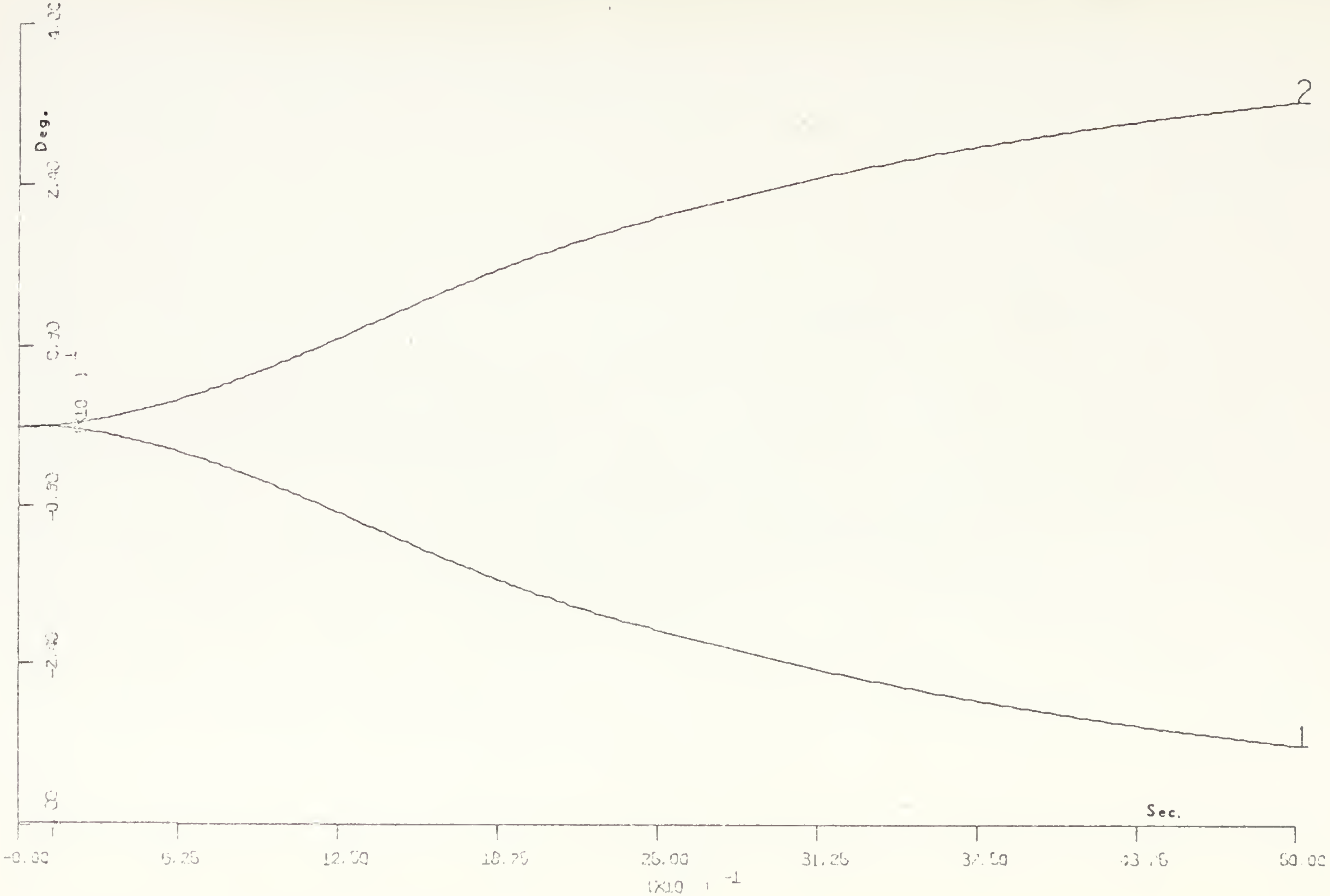


FIGURE II-14. SWAY VS. TIME. TEST #3.

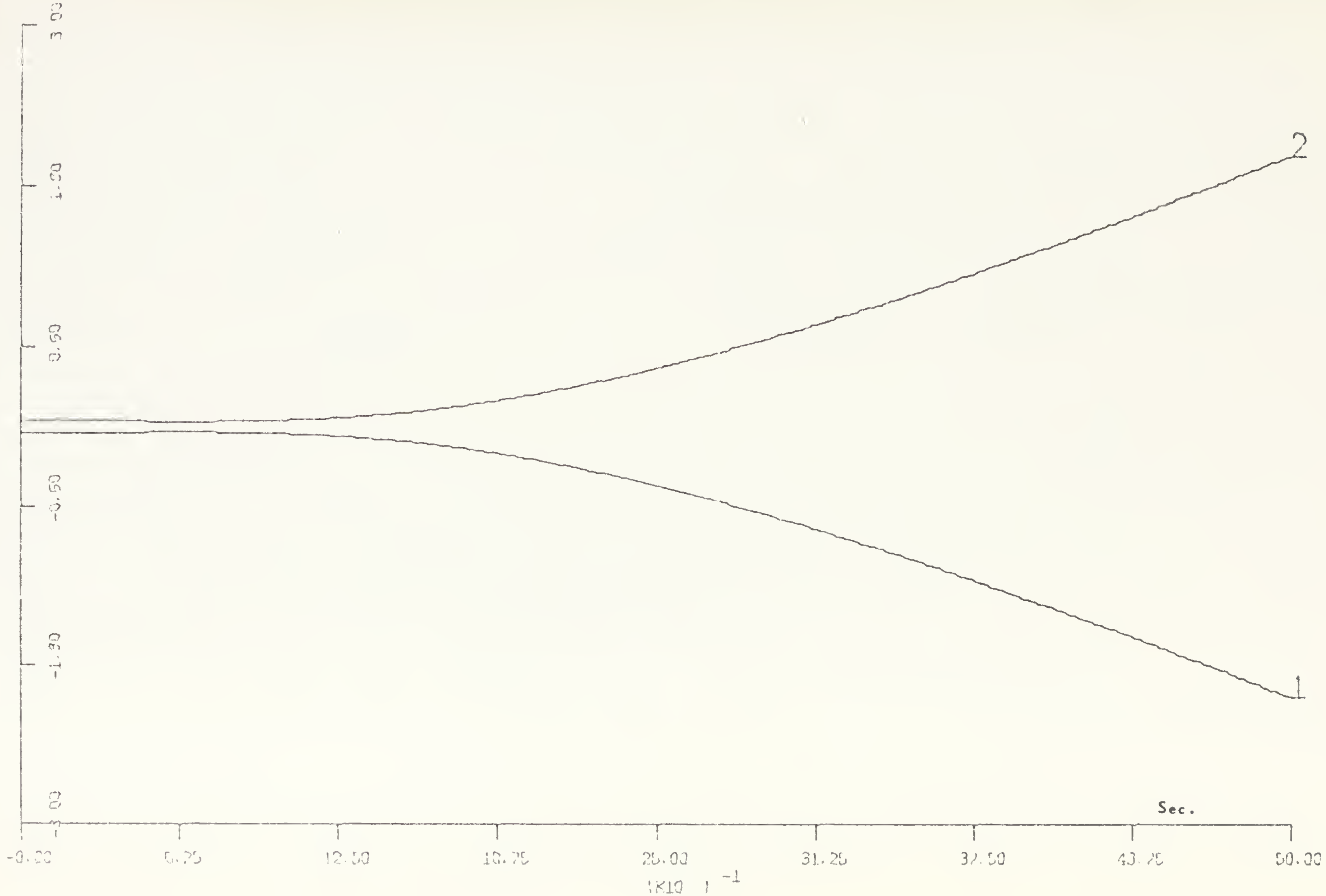


FIGURE II-15. YAW VS. TIME. TEST #3.

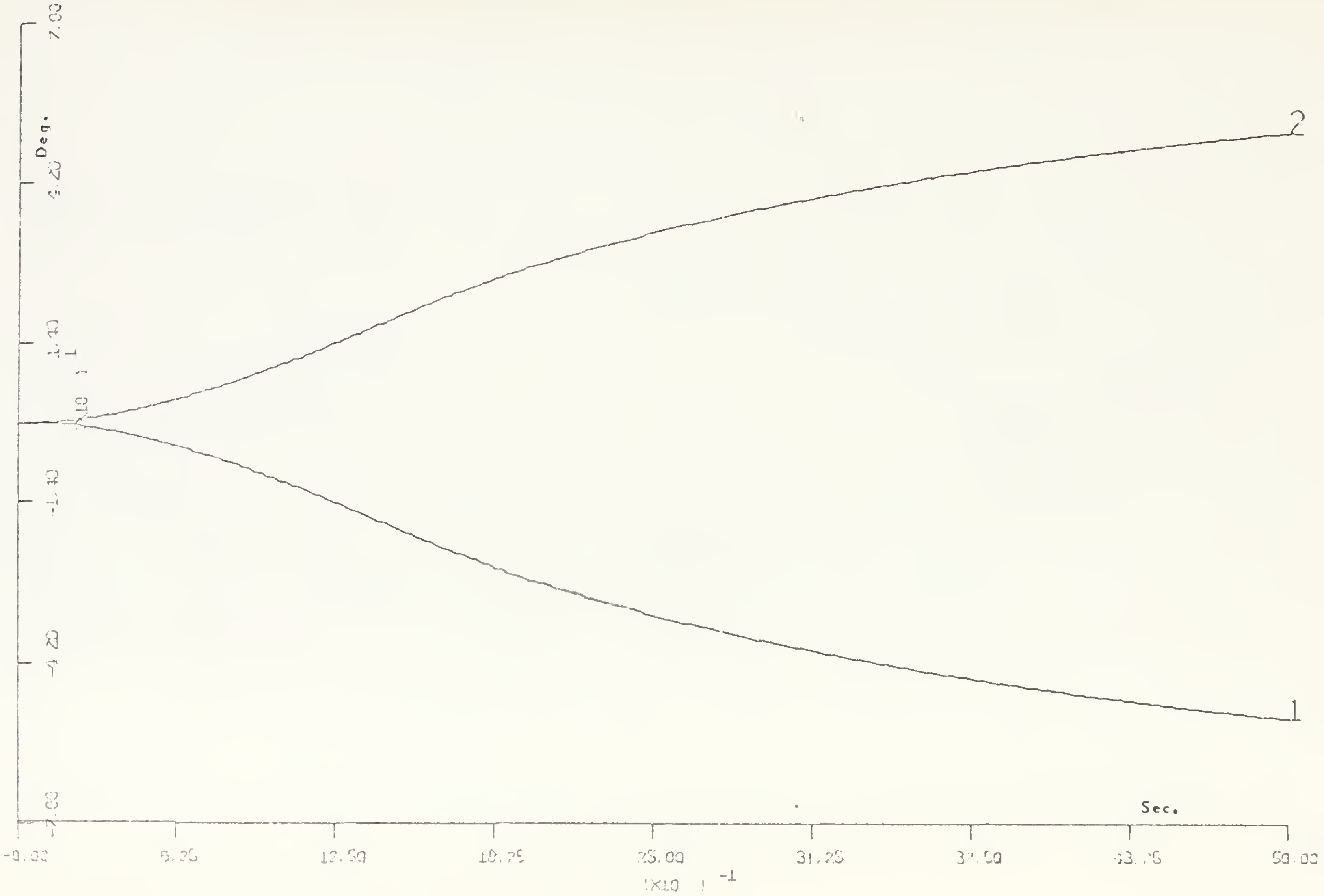


FIGURE II-16. SWAY VS. TIME. TEST #4.

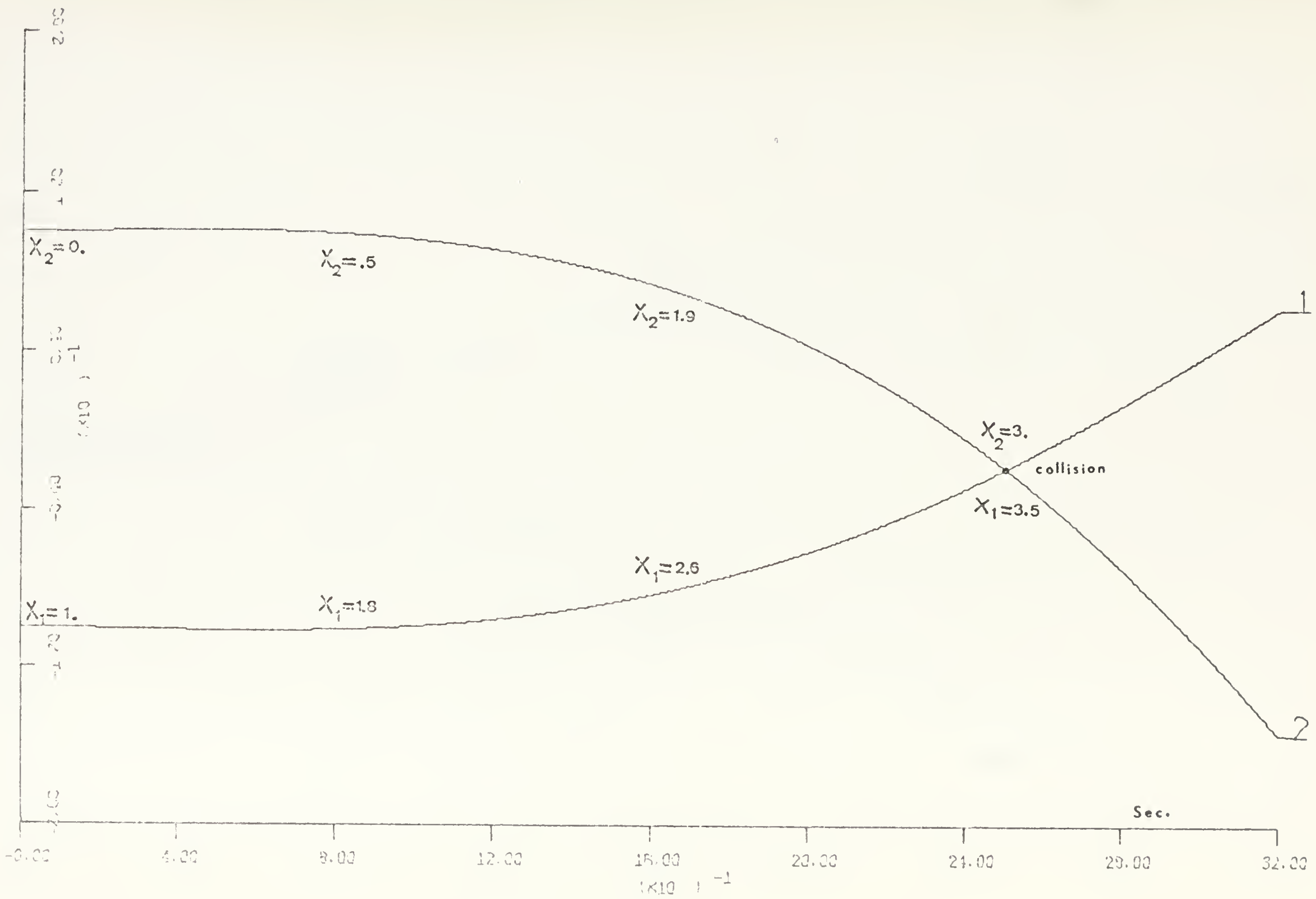
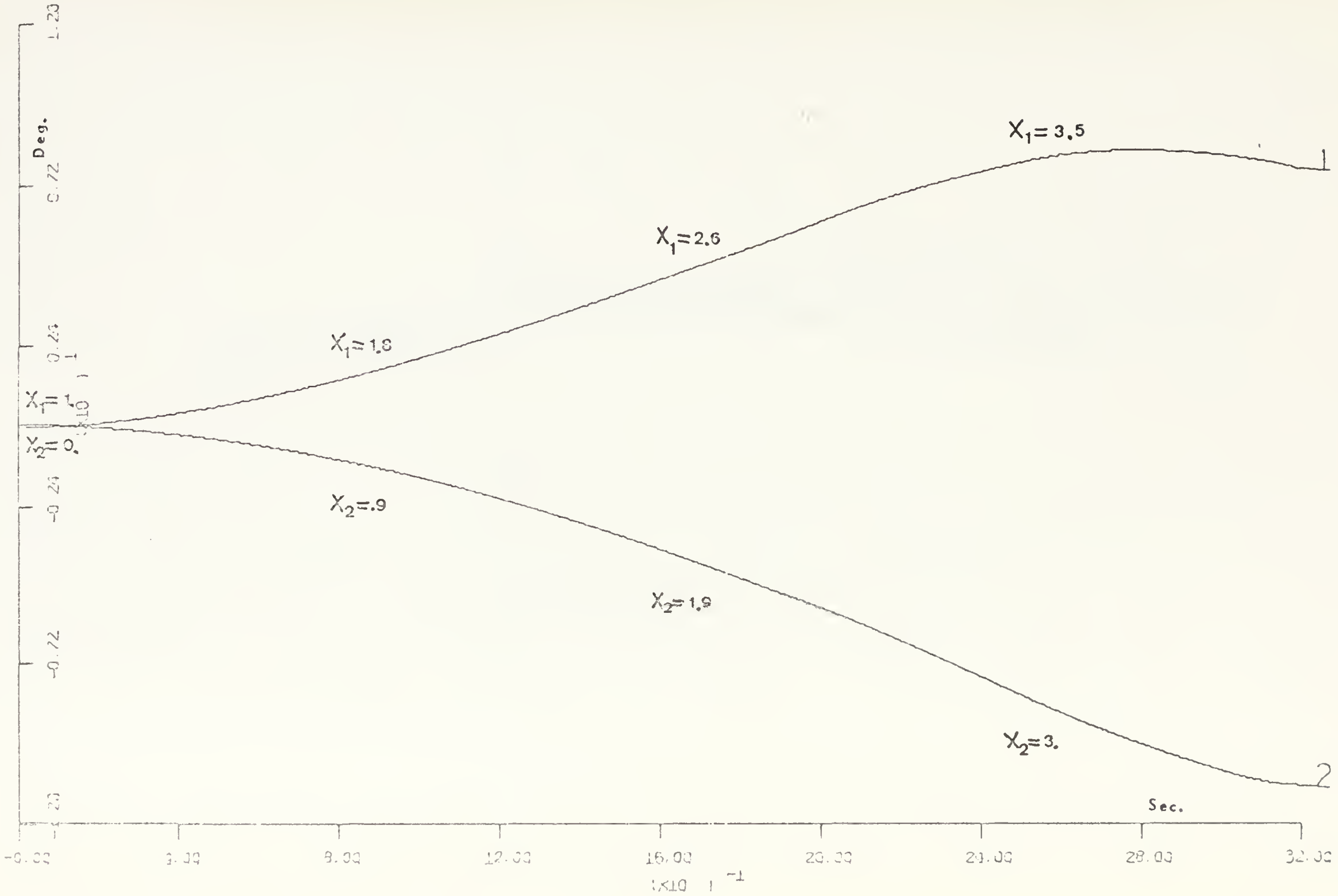


FIGURE II-17. YAW VS. TIME. TEST #4.



Test 4. Collision

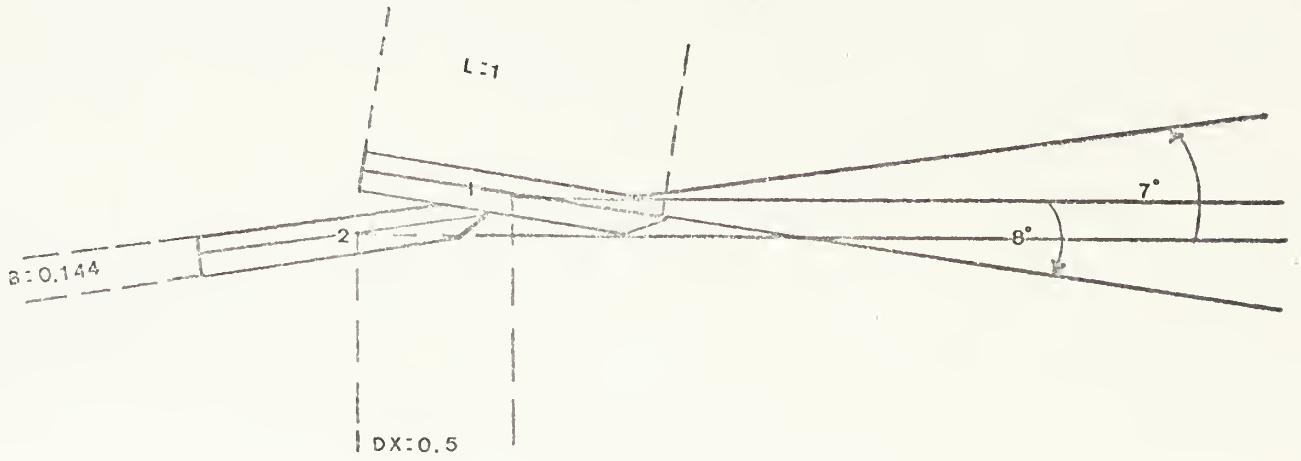


Figure II-18

FIGURE II-19. SWAY VS. TIME. TEST #5.

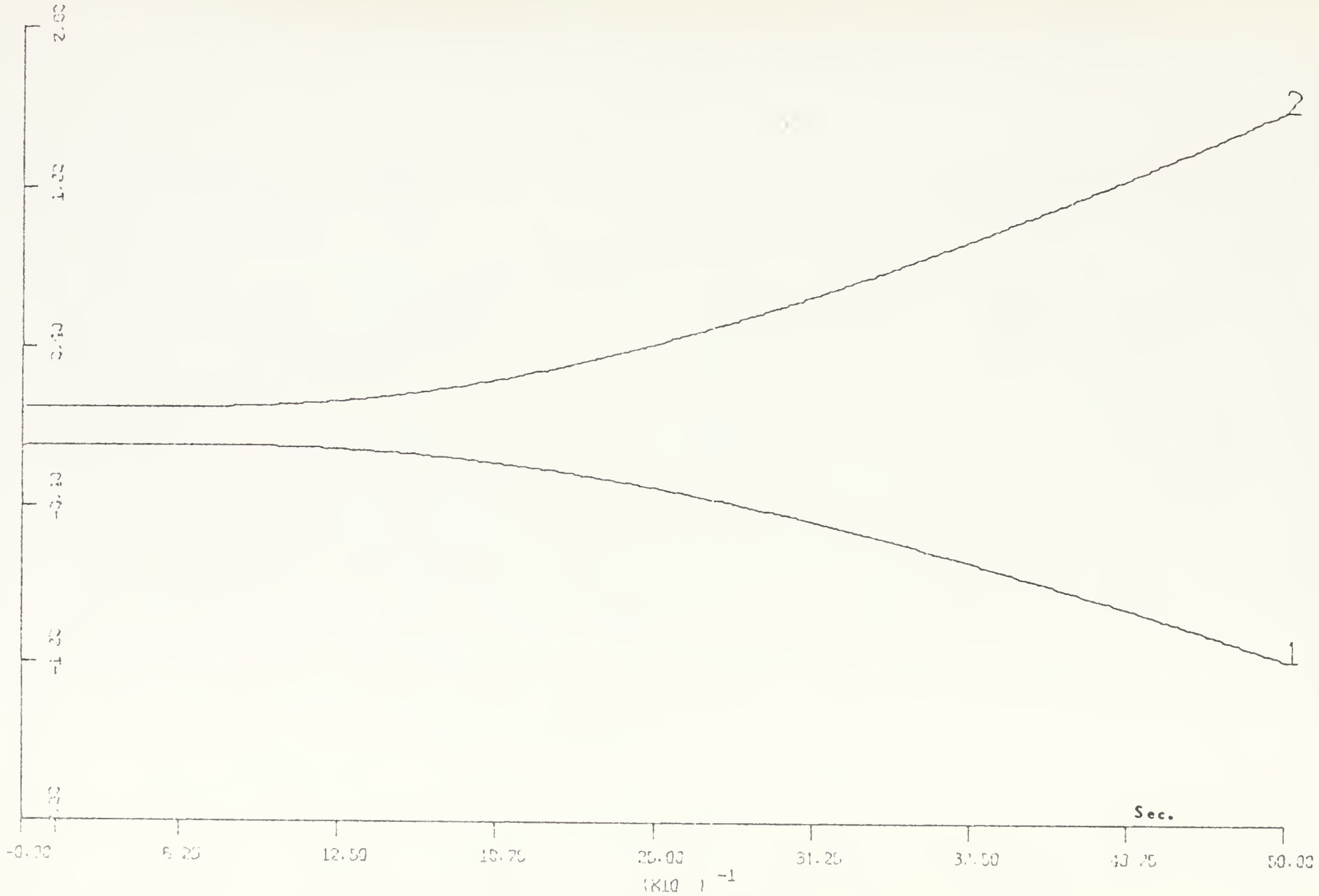
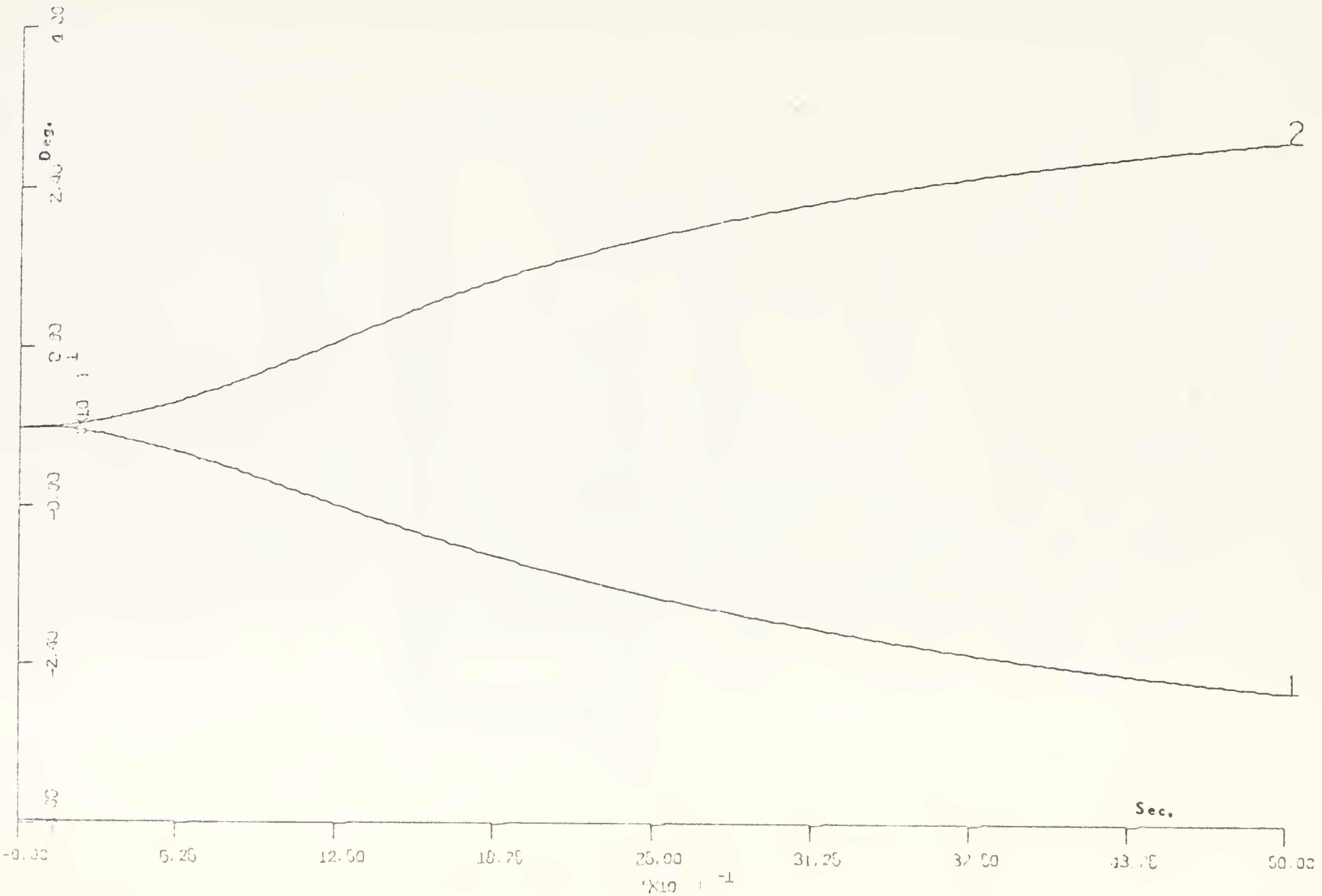


FIGURE II-20. YAW VS. TIME. TEST #5.



G(s) Matrix

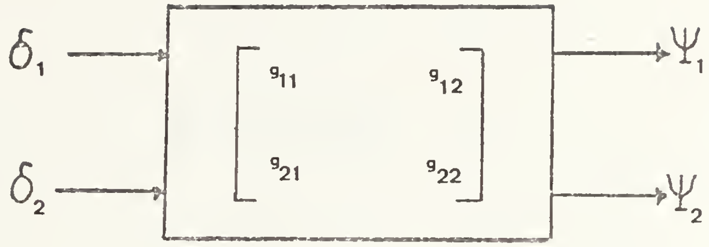


Figure III-1

Closed loop block diagram

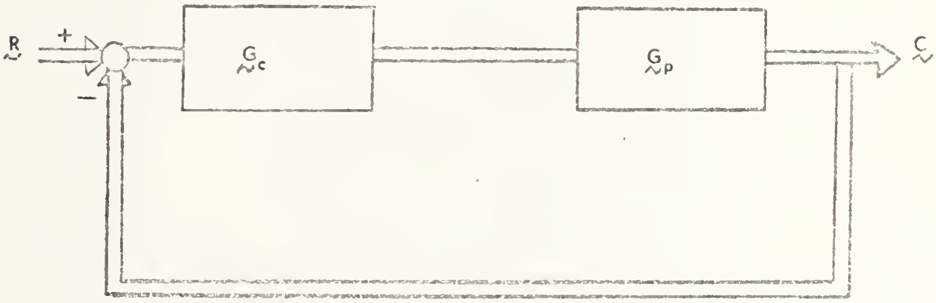
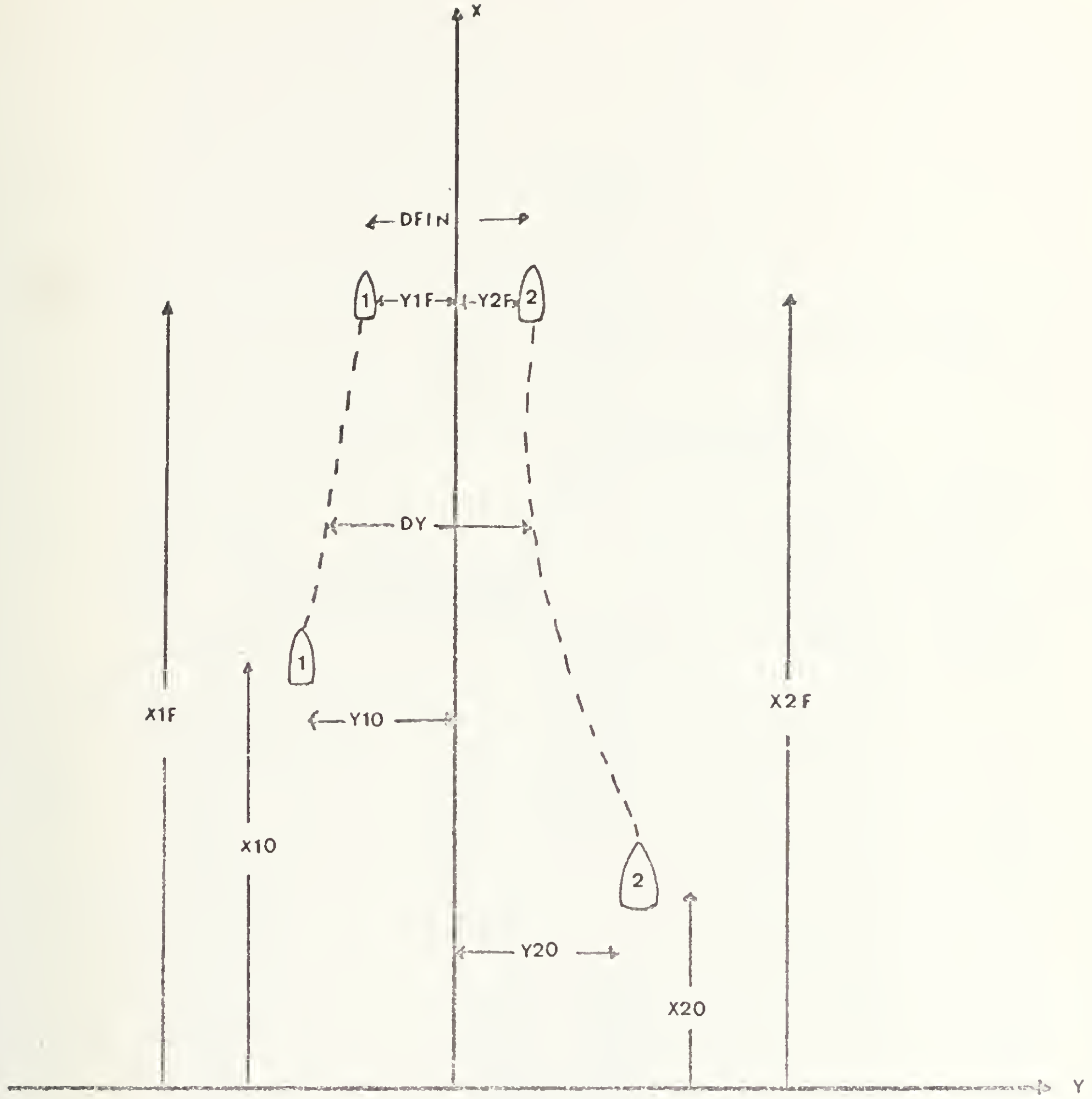


Figure III-2



Replenishment Maneuver

Figure III-3

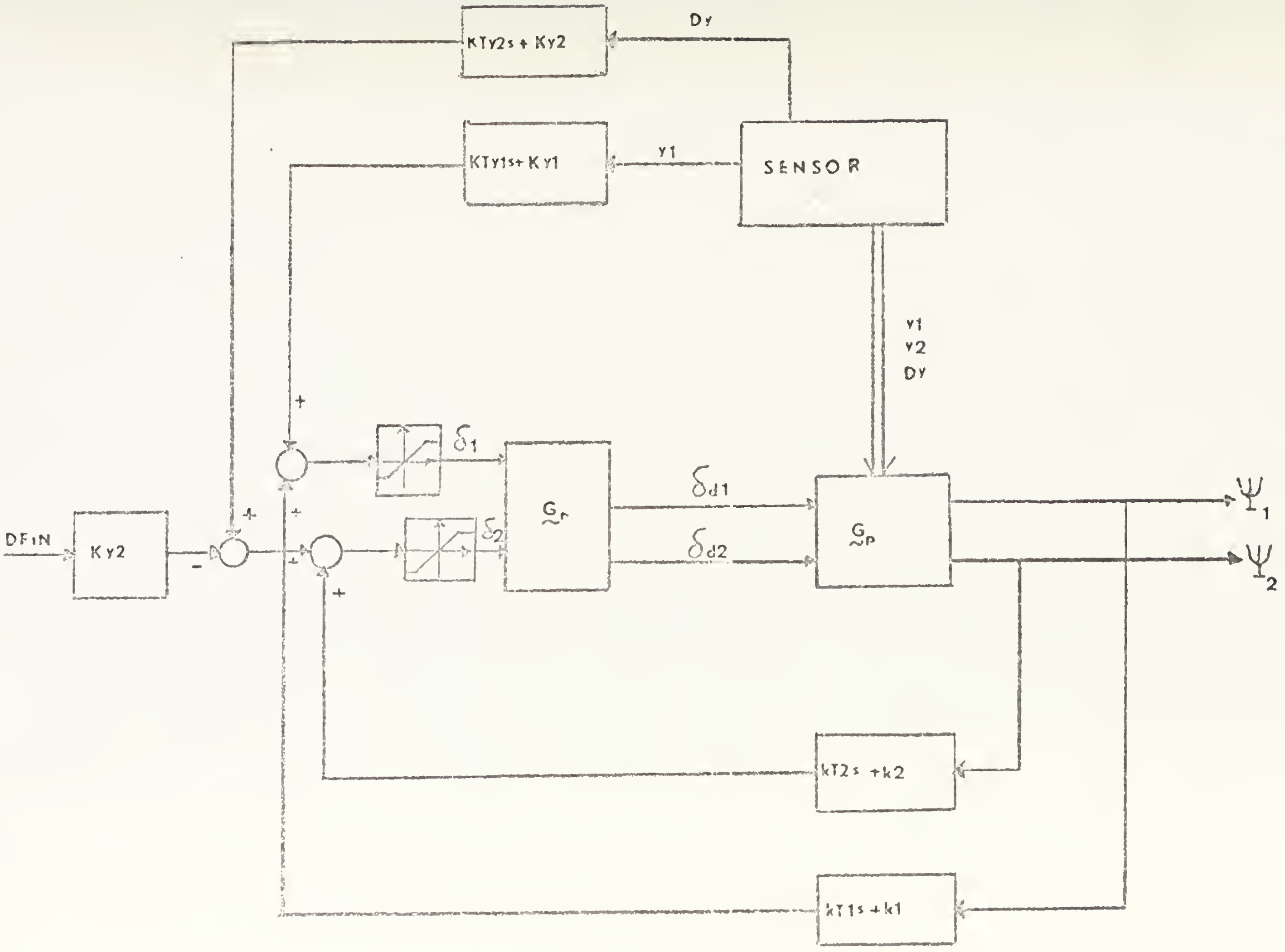


Figure III-4 The Control Loops

FIGURE III-5. SWAY VS. TIME. CONTROLLED PLANT.

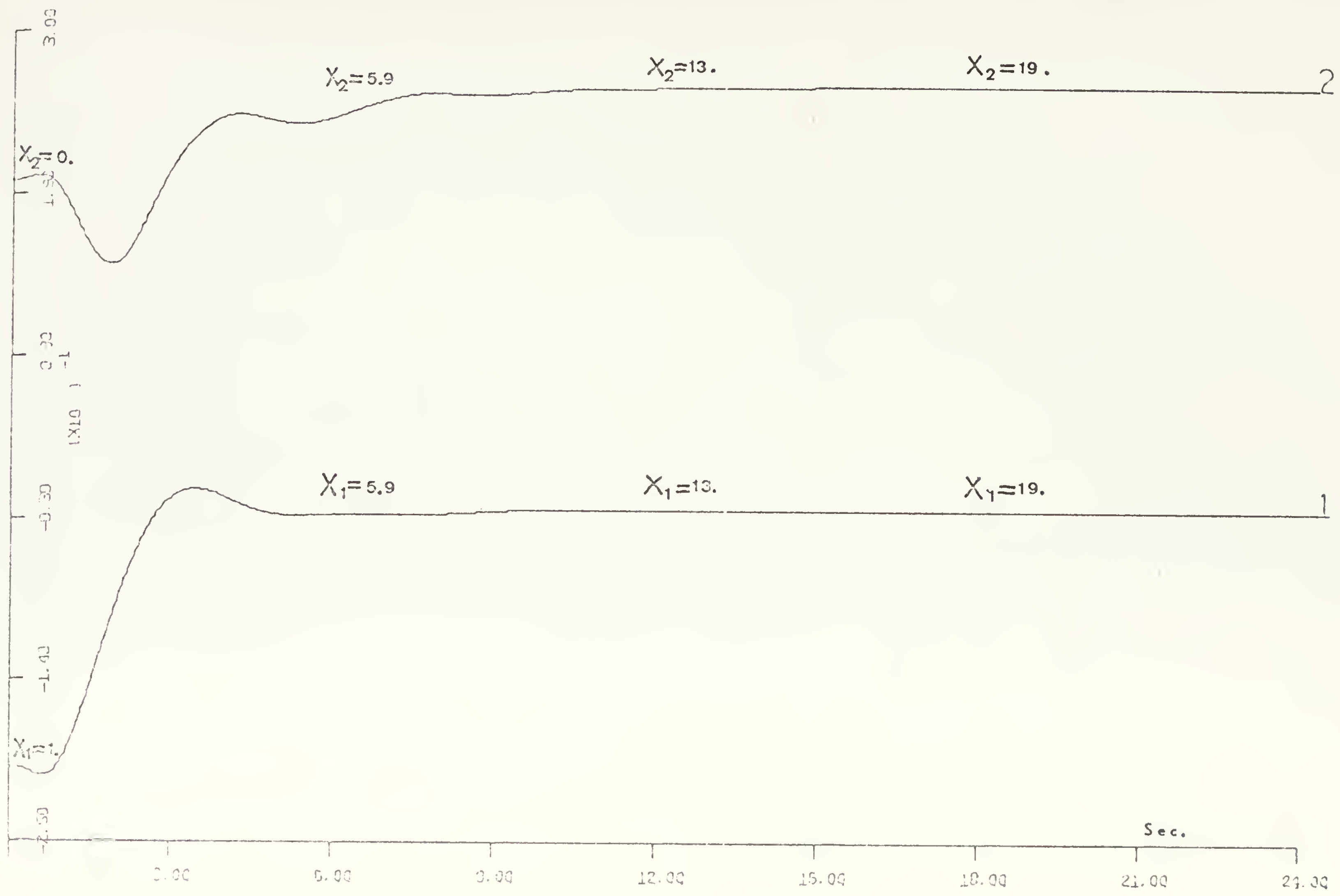


FIGURE III-6. YAW VS. TIME. CONTROLLED PLANT.

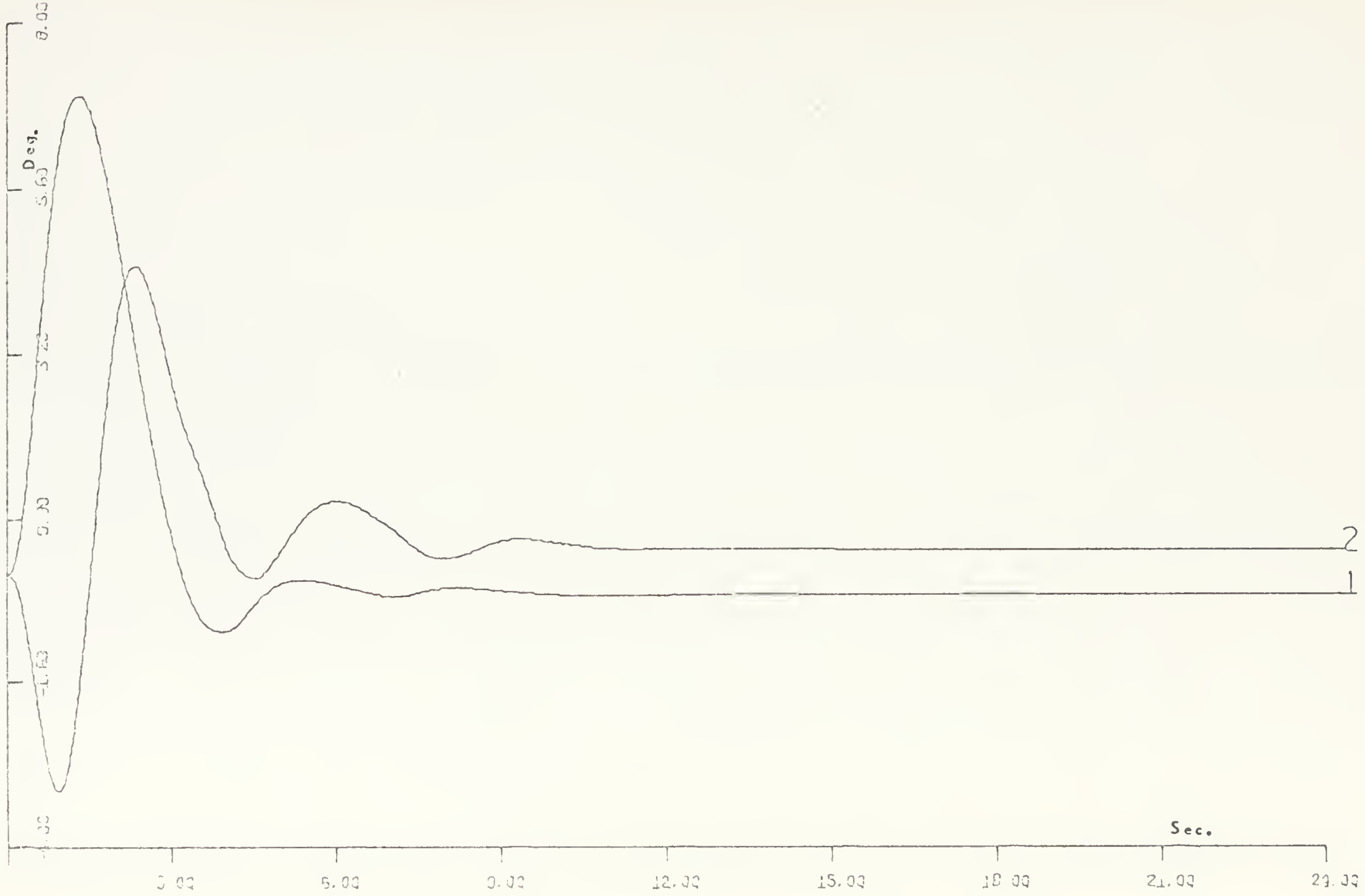
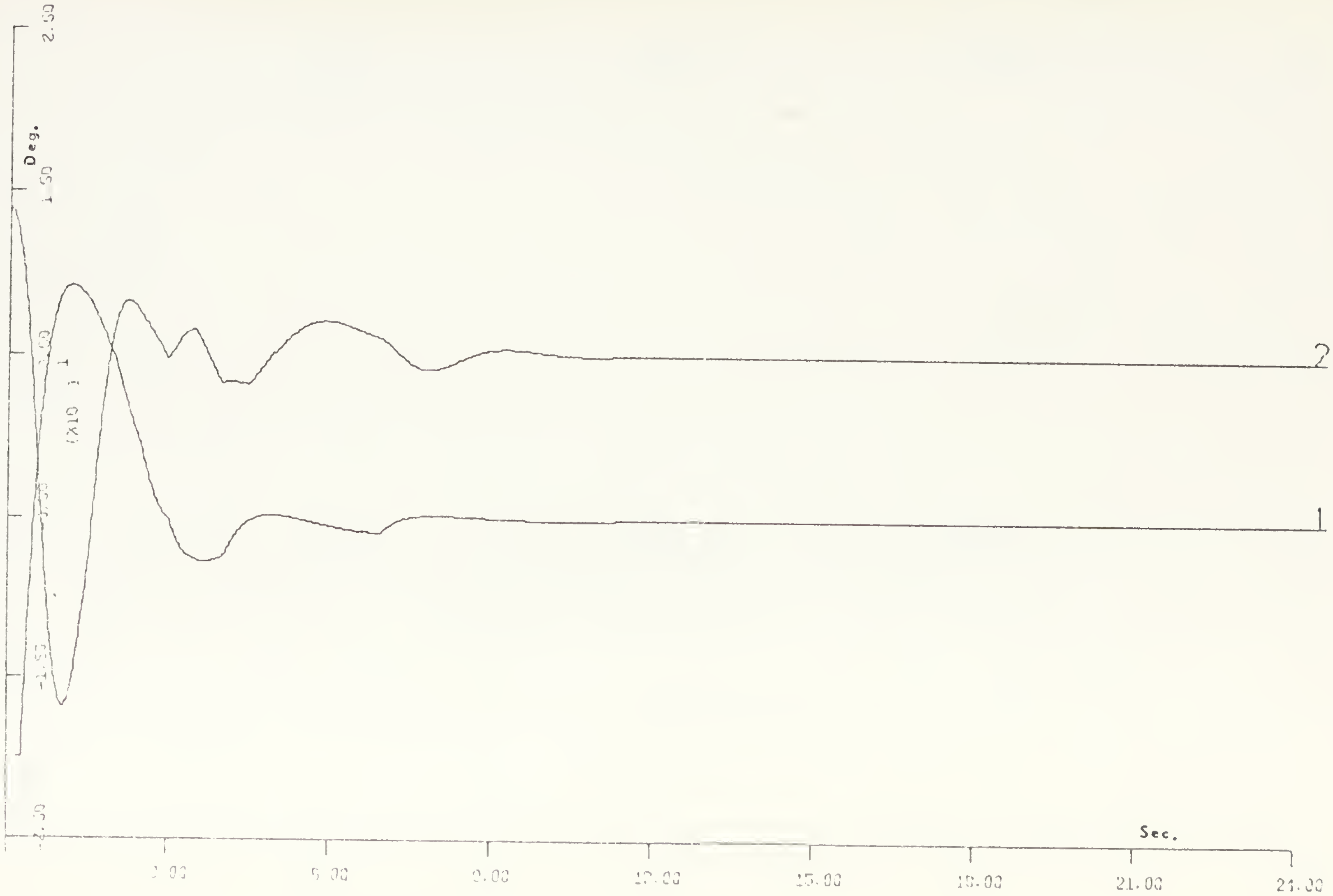


FIGURE III-7. RUDDER VS. TIME. CONT. PLANT.



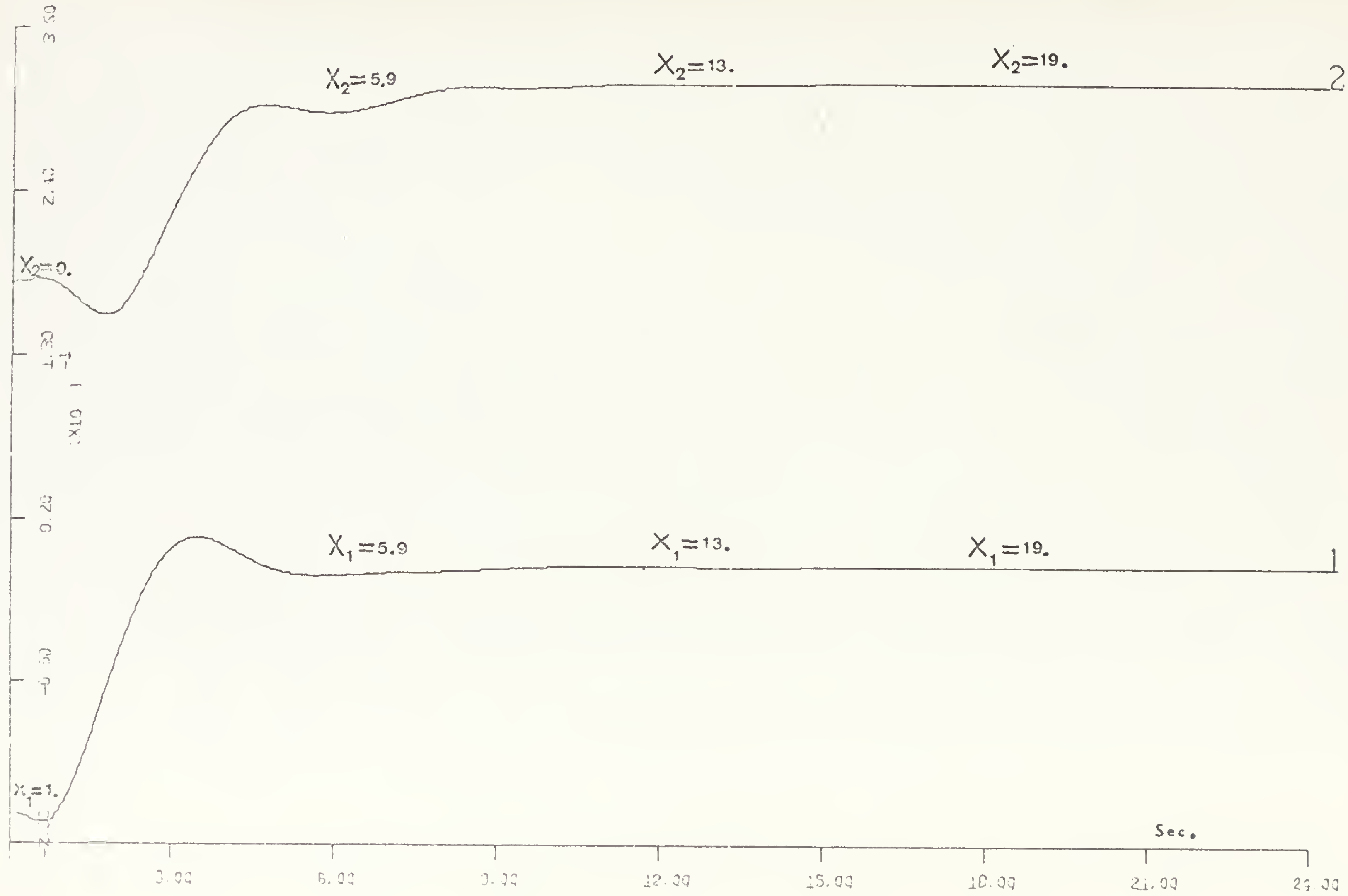


FIGURE III-8. SWAY VS TIME. CONTROLLED PLANT.

FIGURE III-9. YAW VS. TIME. CONTROLLED PLANT.

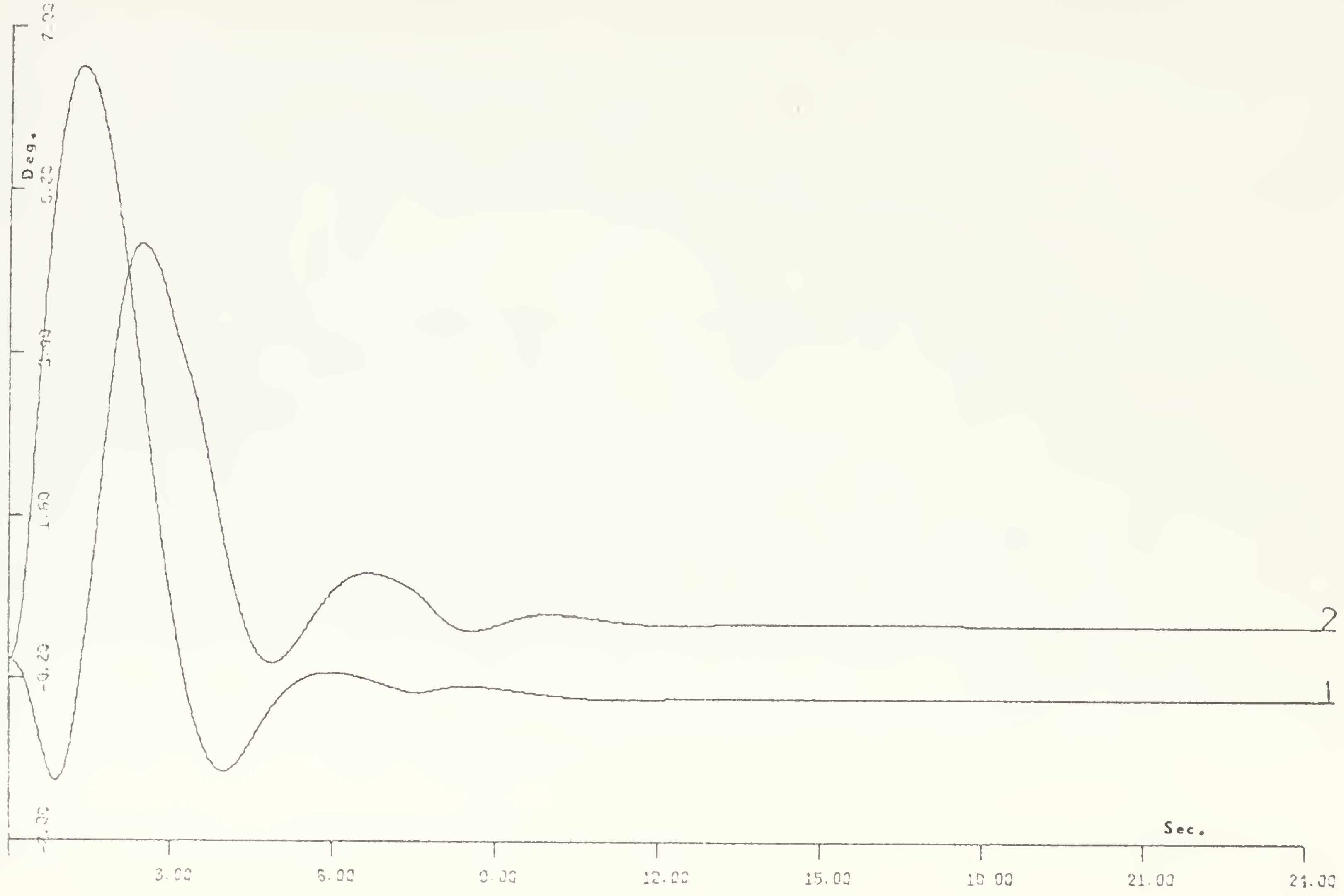
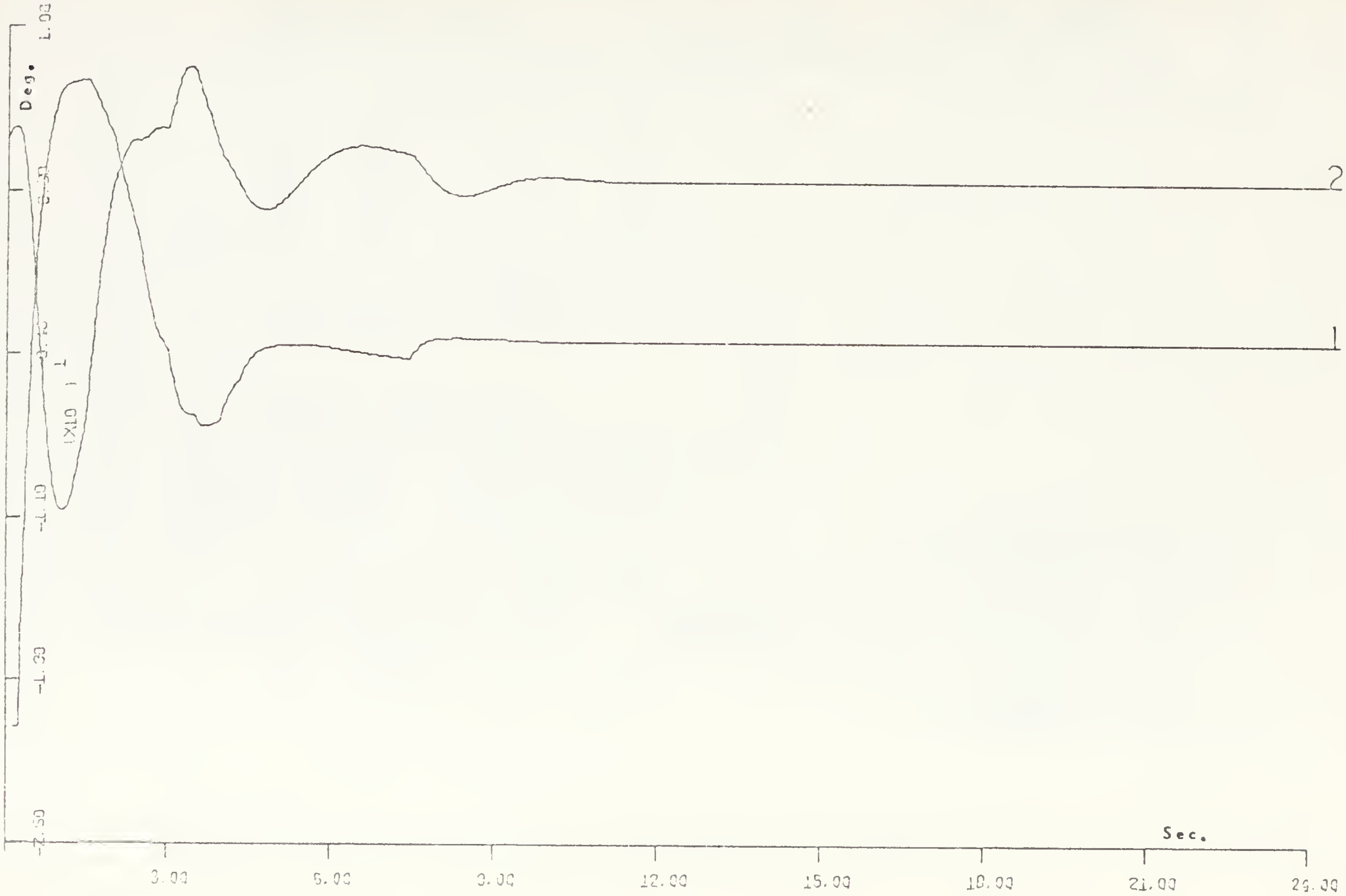


FIGURE III-10. RUDDER VS. TIME. CONT. PLANT.



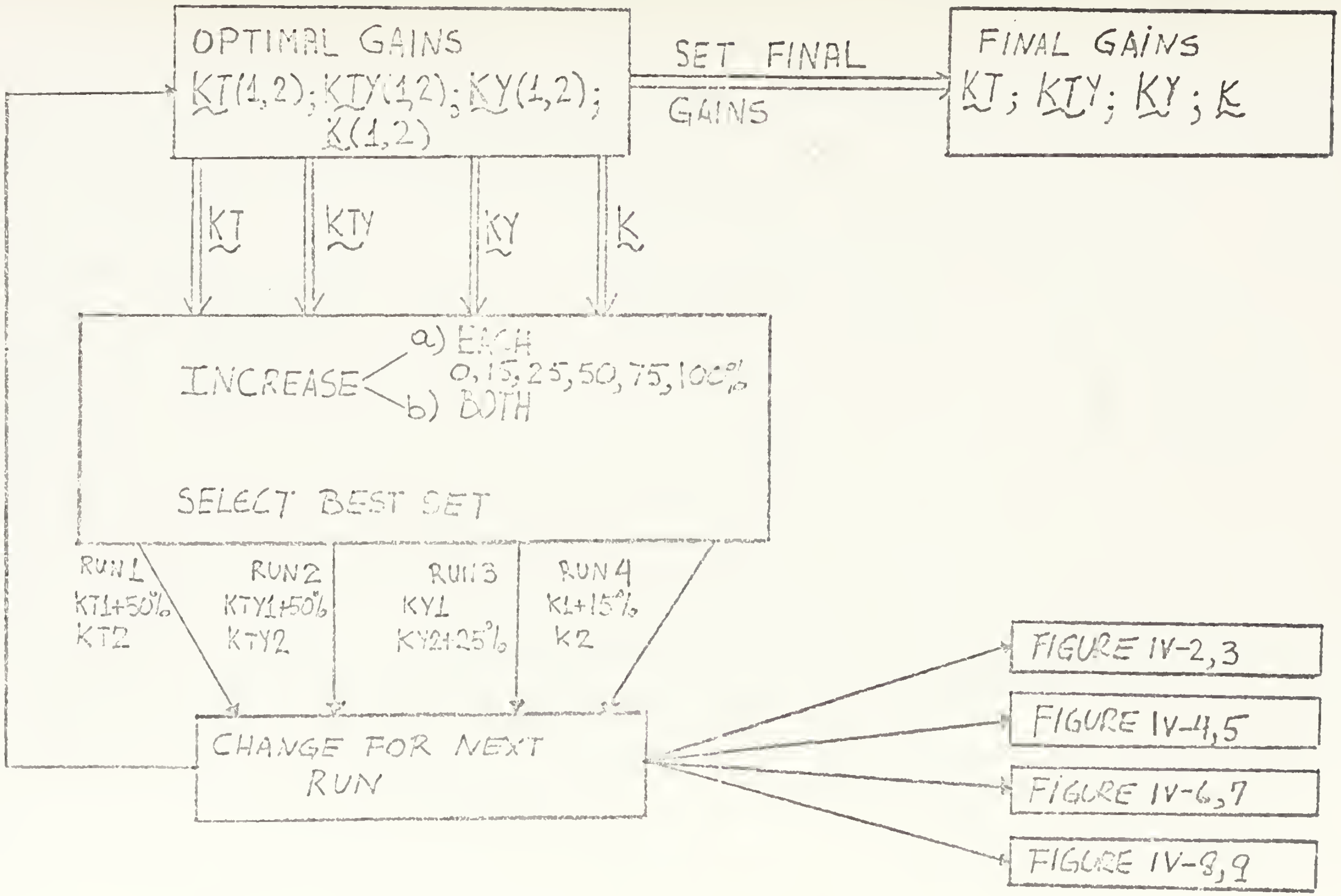


Figure IV-1. Trial and Error Procedure Flow Graph.

FIGURE IV-2. 50% KT1. SWAY VS. TIME.

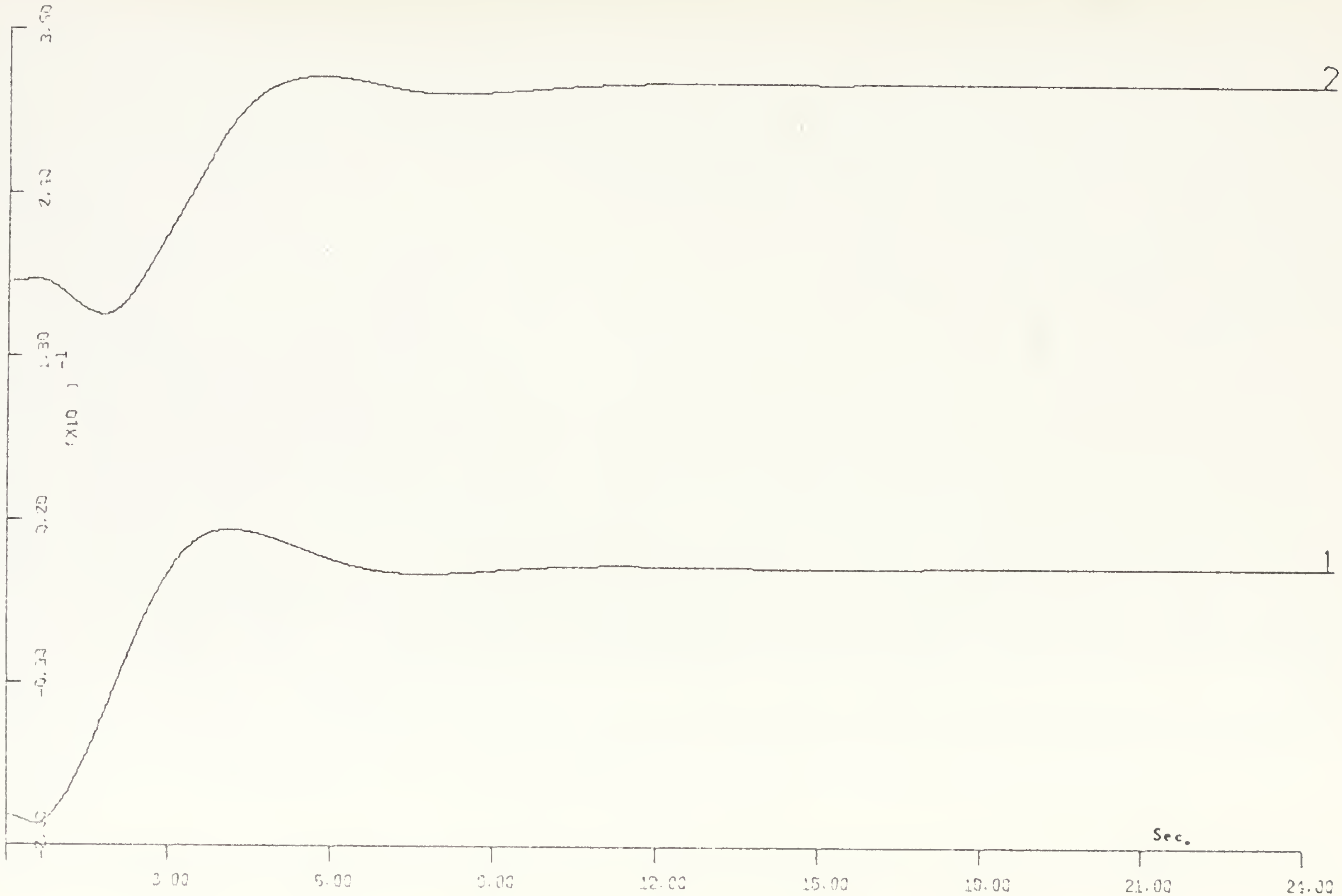


FIGURE IV-3. 50% KT1. YAW VS. TIME.

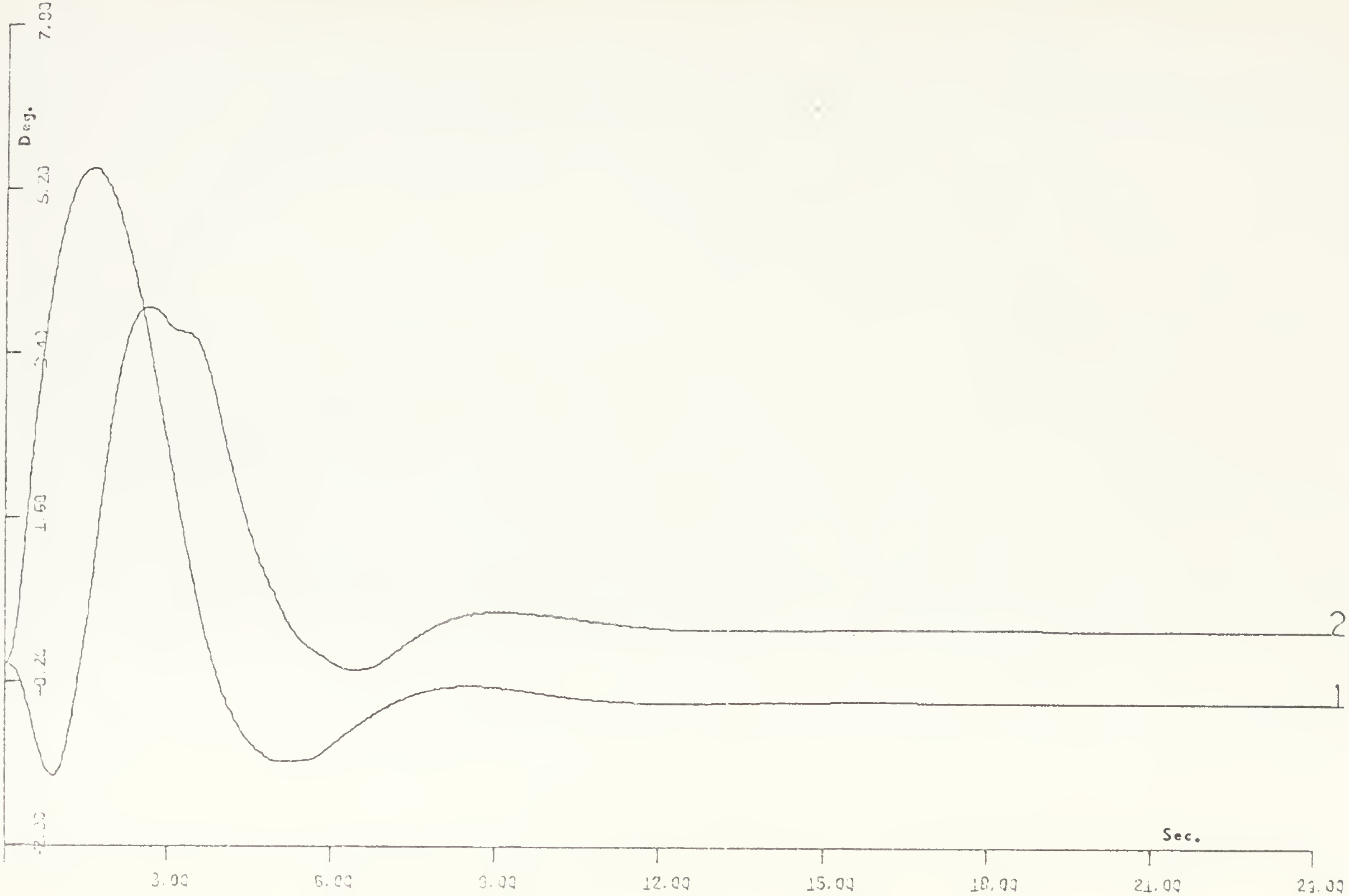


FIGURE IV-4. 50% KTY1. SWAY VS. TIME.

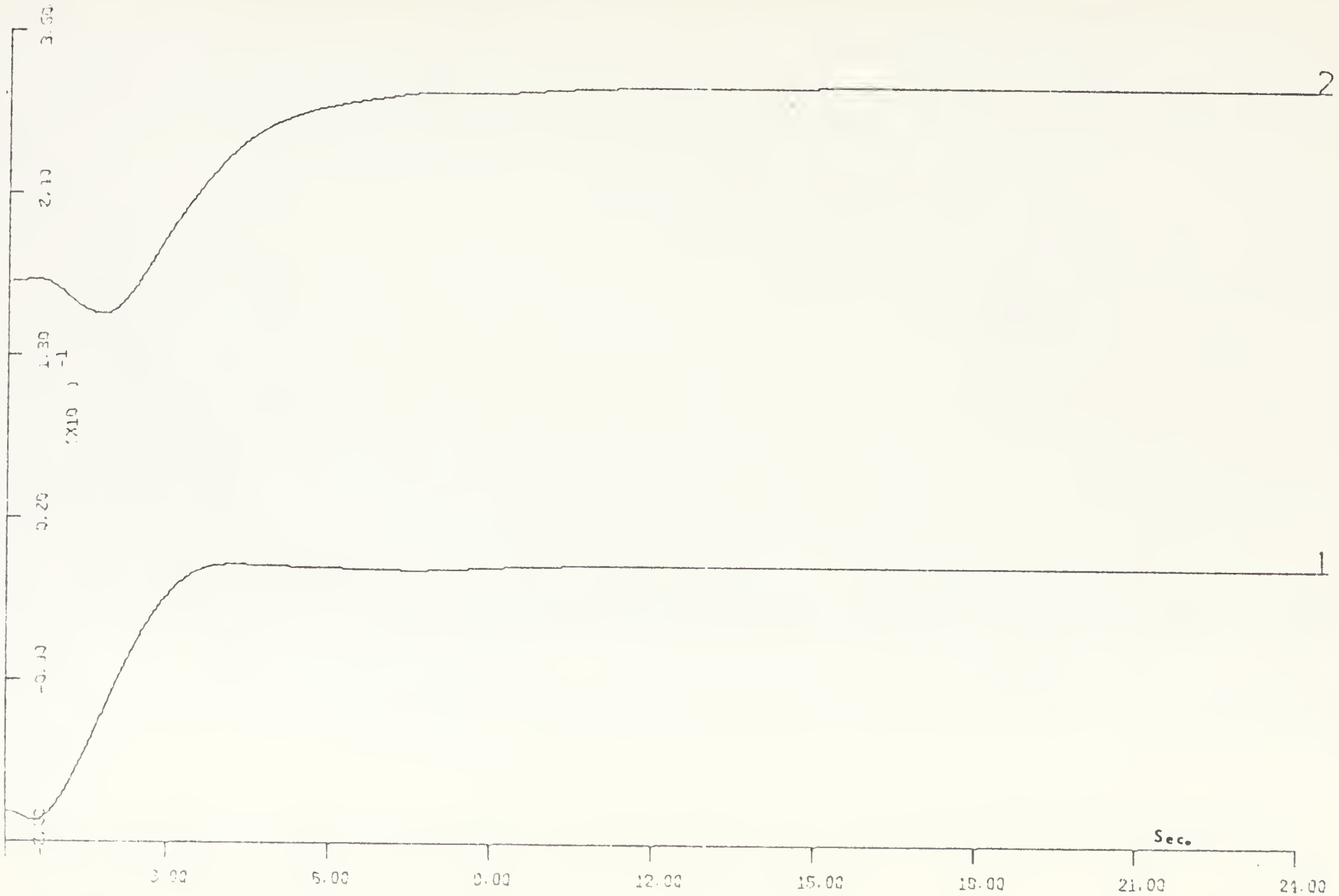


FIGURE IV-5. 50% KTY1. YAW VS. TIME.

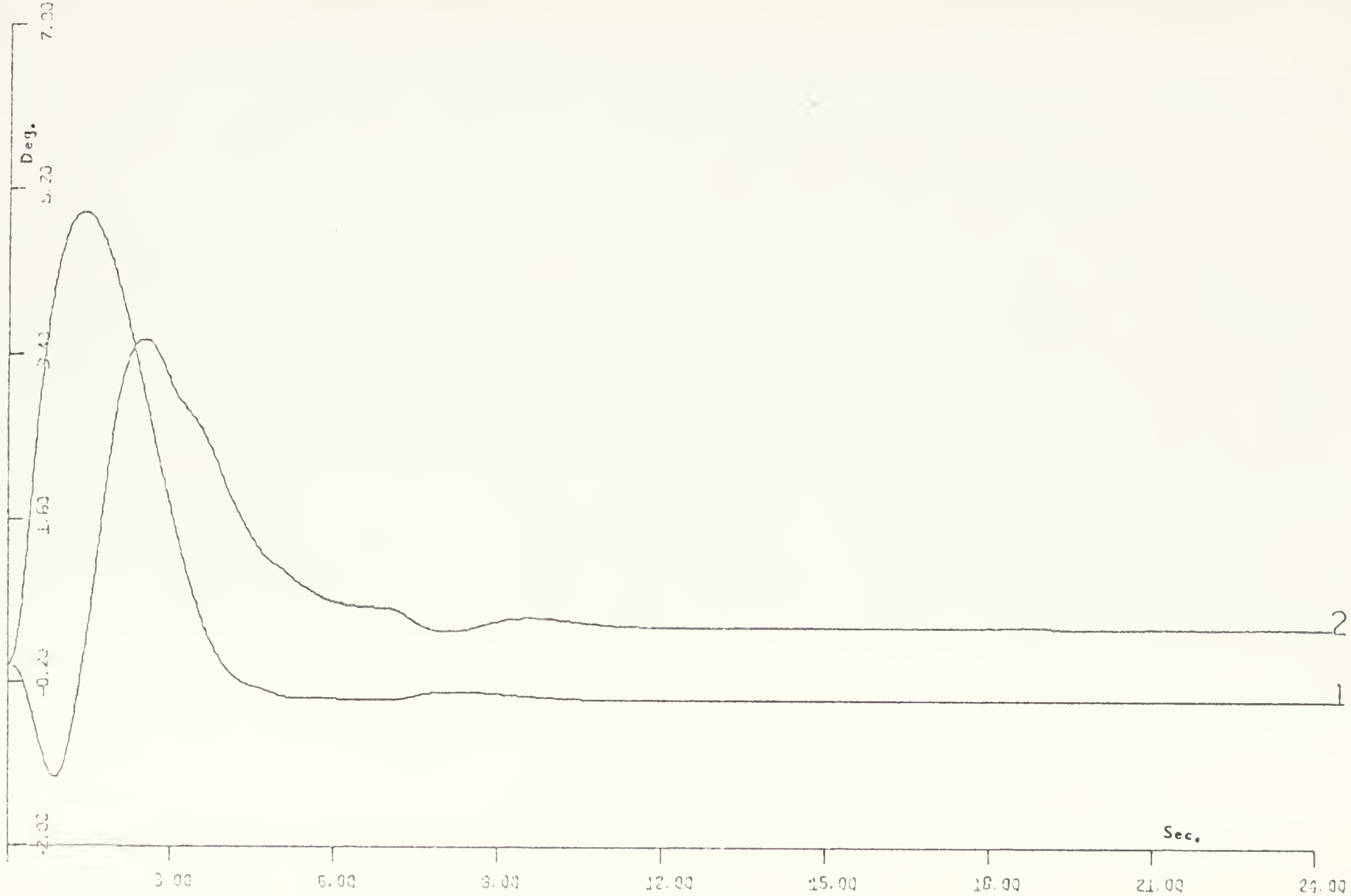


FIGURE IV-6. 25% KY2. SWAY VS. TIME.

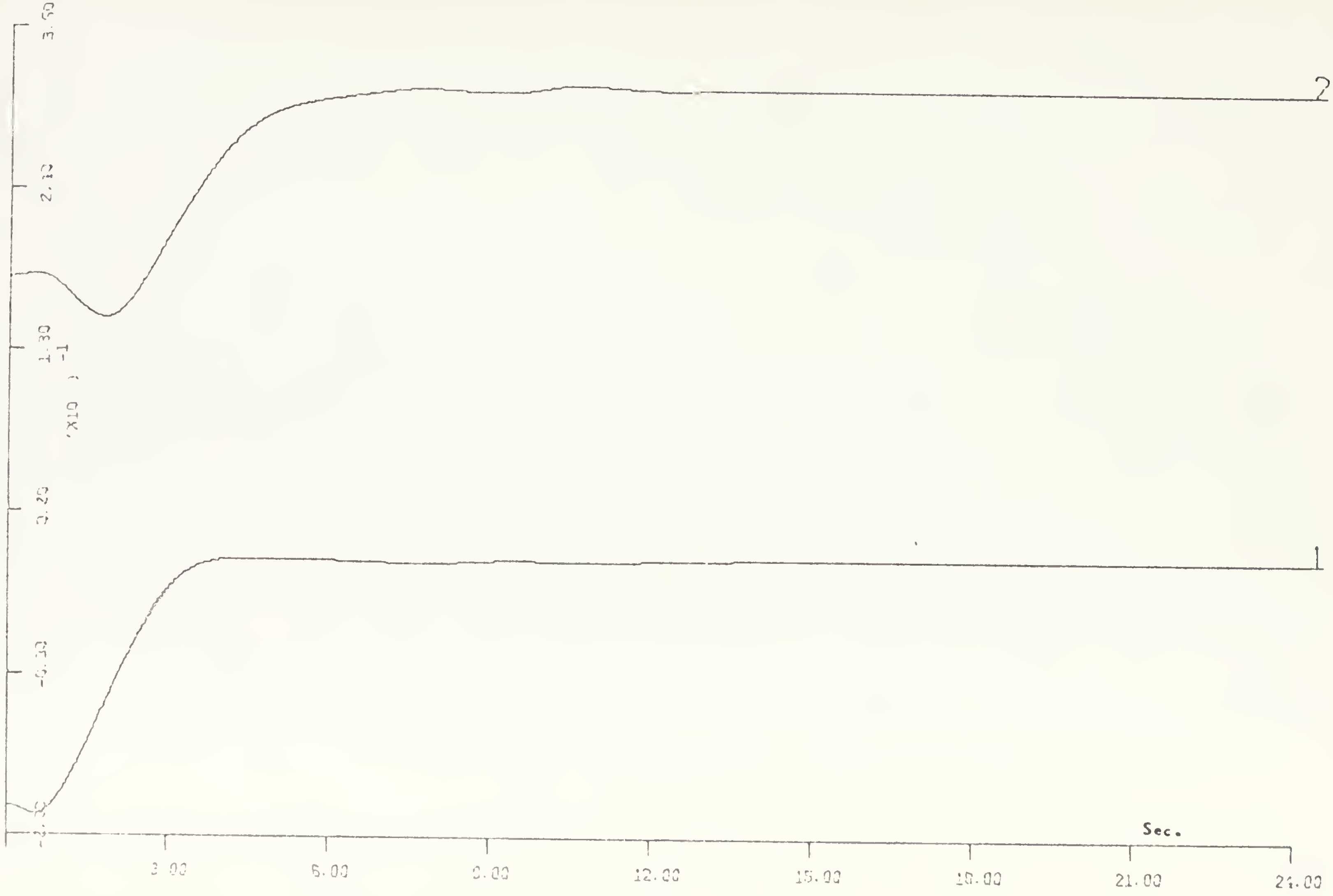


FIGURE IV-7. 25% KY2. YAW VS. TIME.

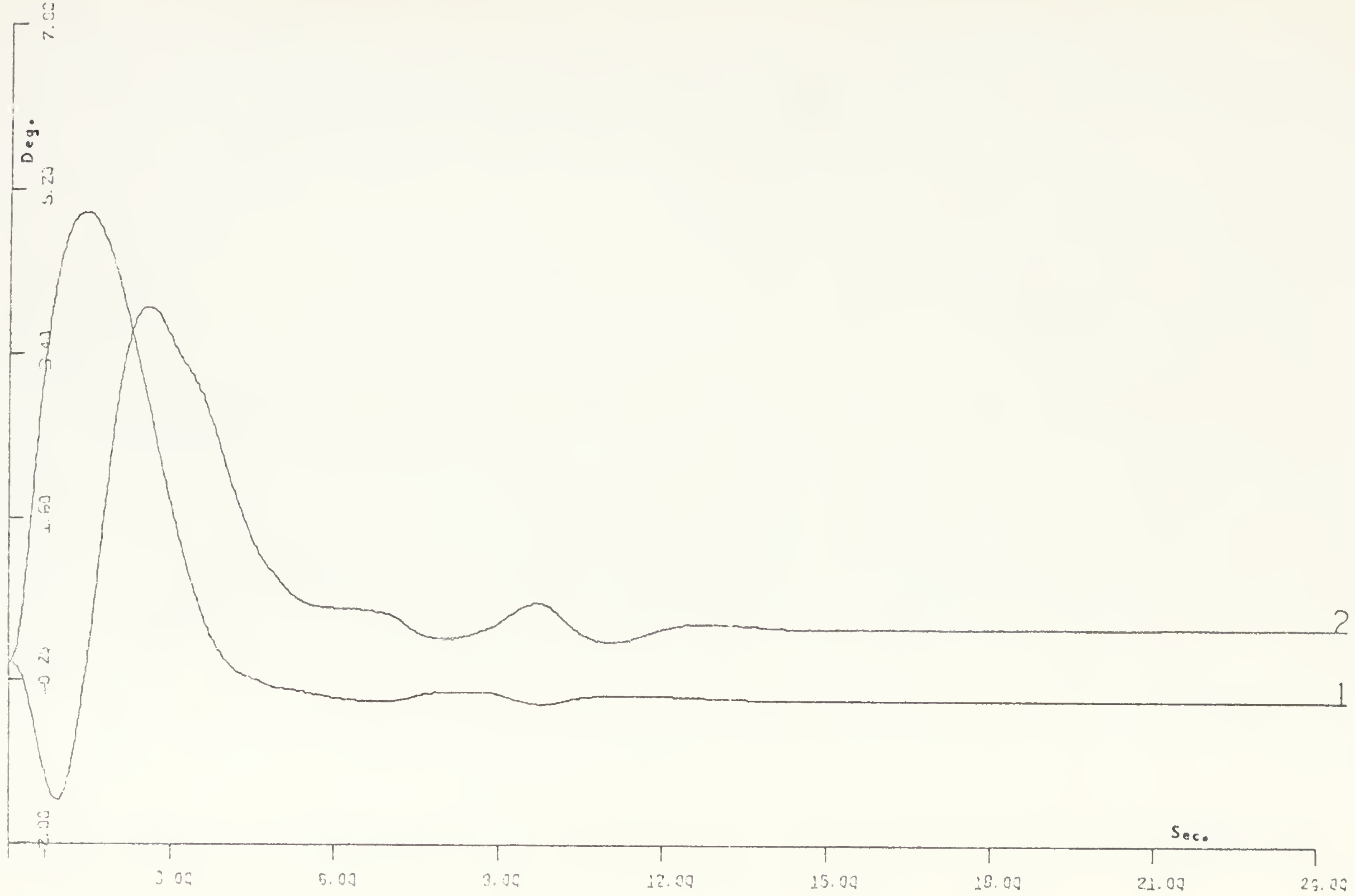


FIGURE IV-8. 15% K1. SWAY VS. TIME.

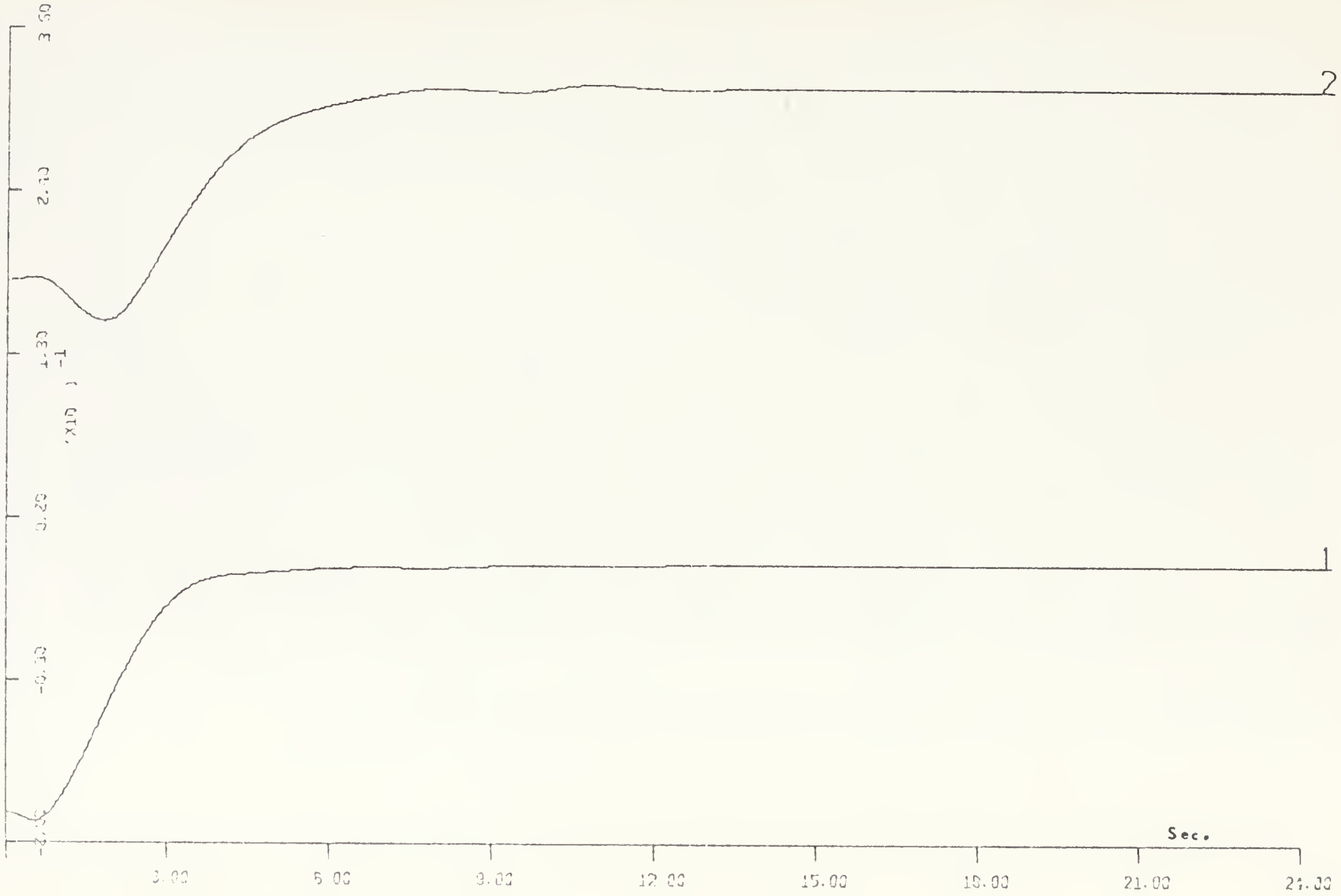
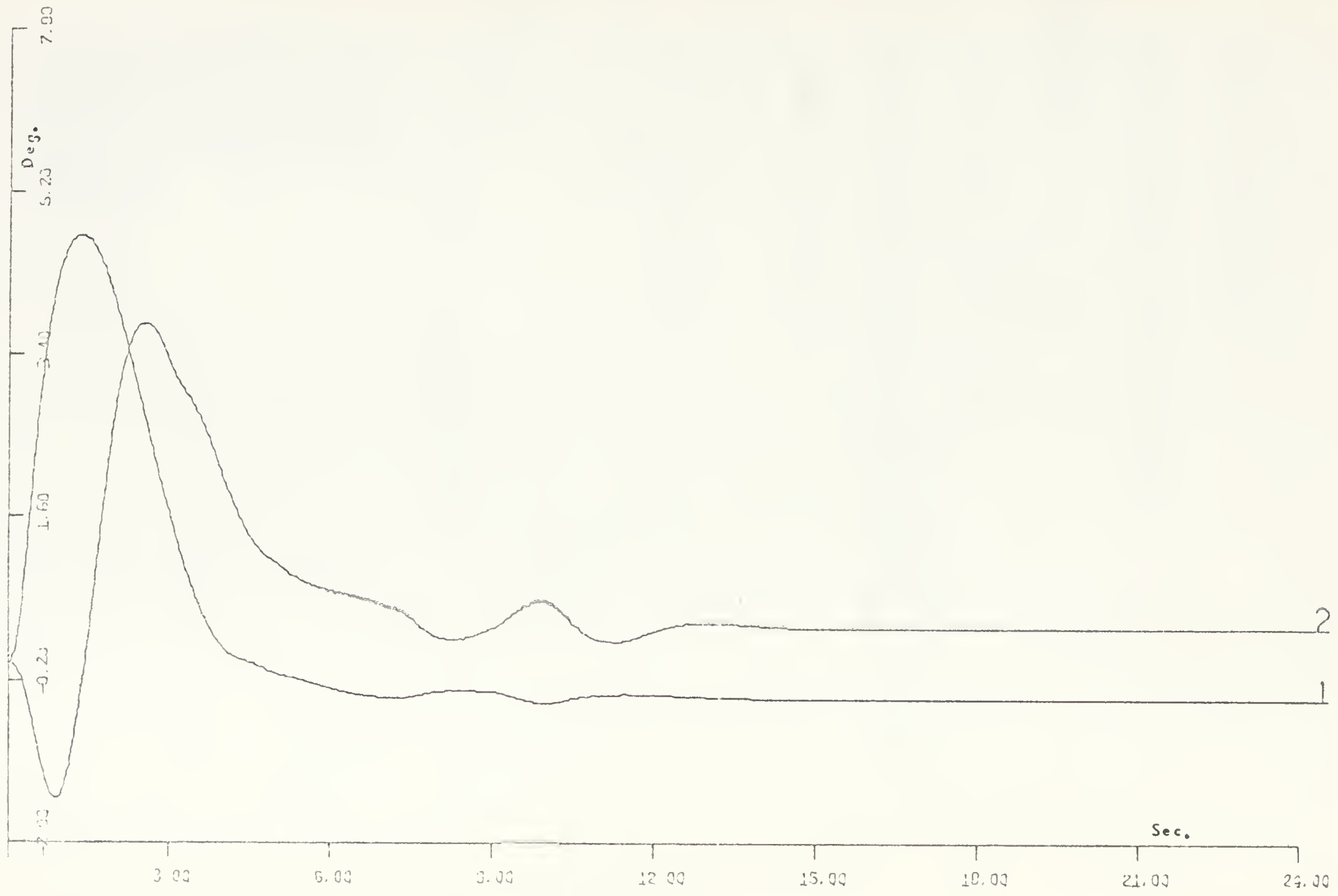


FIGURE IV-9. 152 K1. YAW VS. TIME.



```
//ASTOZ JOB (1025,0732,EA32),'ASTORQUIZA',TIME=4
// EXEC DSL
//DSL.INPUT DD *




*       COMPUTER PROGRAM I



*       LINEAR RESPONSE OF THE MARINER


INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=1

*       HYDRODYNAMIC COEFFICIENTS

CONST NR=-0.00227,NV=-0.00351,NVD=-0.000197
CONST MYVD=0.015,MYR=0.0051,IZNRD=0.00078,MXUD=0.0085
CONST YV=-0.01243,XU=-0.0012,YRD=-0.0027
CONST YDELR=-0.0027,NDELR=-0.00126,XN=.00005

*       RUDDER DEFLECTION

PARAM D1=0.1

*       INITIAL CONDITIONS

INCON X0=0.,Y0=0.

INITIAL

PARAM UD1=1.

*       CALCULATION OF THE COEFFICIENTS

           A11=MYVD
           B11=-YV
           A21=-YRD
           B21=MYR
           A12=-NVD
           B12=-NV
           A22=IZNRD
           B22=-NR
           A33=MXUD
           B33=-XU
           NC=-XU
           KA1=-YDELR
           KB1=NDELR
           KC1=26.*XN
           D=A11*A22-A12*A21
           IF1=KA1*D1
           IF2=KB1*D1
           IF3=KC1*UD1
```

```
DERIVATIVE

*      TIME DOMAIN SIMULATION

         I1=-B11*ADOT-B21*BDOT+IF1
         I2=-B12*ADOT-B22*BDOT+IF2
         I3=-B33*CDOT+IF3
         ADDOT=(I1*A22-I2*A21)/D
         BDDOT=(I2*A11-I1*A12)/D
         CDDOT=I3/A33
         ADOT=INTGRL(0.,ADDOT)
         BDOT=INTGRL(0.,BDDOT)
         CDOT=INTGRL(1.,CDDOT)
         A=INTGRL(0.,ADOT)
         B=INTGRL(0.,BDOT)
         C=INTGRL(0.,CDOT)
         XDOT=CDOT*COS(B)-ADOT*SIN(B)
         YDOT=CDOT*SIN(B)+ADOT*COS(B)
         X=INTGRL(X0,XDOT)
         Y=INTGRL(Y0,YDOT)
         YAW=B
         SWAY=Y
         SURGE=X
SAMPLE
CONTRL   FINTIM=30.,DELT=0.04,DELS=0.04
PREPAR   1.,SURGE,SWAY,YAW,USPEED
GRAPH  TIME,YAW
GRAPH  TIME,SWAY
GRAPH  TIME,USPEED
GRAPH  SAME,10,10,SURGE,SWAY,USPEED
PRPLOT  ONLY
         CALL  DRWG(1,1,TIME,YAW)
         CALL  DRWG(2,1,SURGE,SWAY)
TERMINAL
         CALL  ENDRW(NPLOT)
END
STOP
//PLOT.STEPLIB  DD  DSN=SYS3.DSLPLOT,UNIT=2321,VOL=SER=CEL00
//PLOT.SYSIN DD *
```

```
//AST02 JOB (1025,0732,EA32),'ASTORQUIZA',TIME=4
// EXEC FORTCLGP,REGION.GO=150K
//FORT.SYSIN DD *
C
C
C
C
C
C
C     COMPUTER PROGRAM II
C
C
C
C
C     THIS PROGRAM WILL PLOT A ROOT LOCUS FOR A CHARACTERISTIC EQUATION UP 00020
C     ORDER 30. ROOT LOCUS POLES ARE PLOTTED WITH AN X, ROOT LOCUS ZEROS AR00030
C     PLOTTED WITH A SQUARE, AND INTERMEDIATE ROOT POINTS ARE PLOTTED WITH 00040
C     PLUS. THE STARTING VALUE OF ROOT LOCUS GAIN AND THE NUMBER OF DECADES00050
C     TO BE SPANNED BY THE GAIN MUST BE SPECIFIED. THE GRAPH PLOT IS BASED 00060
C     PLOTTING EVERY TENTH POINT AS THE GAIN VARIES BETWEEN ITS INITIAL AND00070
C     FINAL VALUE IN 300 STEPS.                                            00080
C           THE DATA CARDS ARE SUBMITTED IN THE FOLLOWING MANNER. SUBMIT A 00090
C     COMPLETE SET OF DATA CARDS FOR EACH RUN. MULTIPLE RUNS MAY BE MADE.  00100
C     CARD 1  THE FIRST LINE OF THE GRAPH TITLE  (IN COLUMNS 1'48)         00110
C     CARD 2  THE SECOND LINE OF THE GRAPH TITLE  (IN COLUMNS 1'48)        00120
C     CARD 3 GRAPH DATA IN 2F10.0,5I10 FORMAT AS FOLLOWS
C     WORD 1=XSCALE
C     WORD 2=YSCALE
C     WORD 3=IXUP
C     WORD 4=IYRIGH
C     WORD 5=IWIDE
C     WORD 6=IHIGH
C     WCRD 7=IGRID
C     CARD 4 THE ORDER OF THE CHARACTERISTIC EQUATION (I3 FORMAT)          00130
C     CARD 5 CONSTANT PART OF THE COEFFICIENTS IN DESCENDING ORDER(8E10.  F00140
C 5)
C     CARD 6 VARIABLE PART OF THE COEFFICIENTS IN DESCENDING ORDER(8E10.  F00150
C 5)
C     CARD 7 INITIAL VALUE OF THE VARIABLE ( E10.5 FORMAT) , MUST NOT BE   00160
C ZERO
C     CARD 8 NUMBER OF DECADES TO BE SPANNED. (FROM 1'10) , (I3 FORMAT)    00170
      DIMENSION R(129),X(129),IT(10),ROOTR(128),ROOTI(128),                00190
     1A(129),B(129),ROOTJ(128),ROOTM(128),AP(129),AZ(129)                  00200
      COMMON R,VAR,NO,ROOTR,ROOTI,AP,X                                     00210
      REAL*8  LABEL/8H                  /,ITITLE(12)                        0230
      CALL ERRSET (208,256,-1,0,0,207)
  206 MCD=1                                                                 0220
   68 FORMAT(8E15.5)                                                       00240
```

```
   70 FORMAT     (15HOIMAGINARY PART,//)
   69 FORMAT     (10HOREAL PART,//)
  200 FORMAT(6A8)                                                       00270
  203 FORMAT(I3)                                                        00280
  204 FORMAT(2F10.0,5I10)
  208 FORMAT(2F10.1,5I10)
      READ 200,(ITITLE(I),I=1,6)                                        00300
      READ 200,(ITITLE(I),I=7,12)                                       00310
   24 FORMAT(1H1,18H THE INPUT DATA IS)
      PRINT 24                                                          00330
  324 FORMAT(29HOGRAPH TITLE AND SCALING DATA,/)
      PRINT 324                                                         00350
  400 FORMAT(1X,6A8)
      PRINT 400,(ITITLE(I),I=1,6)                                       00360
      PRINT 400,(ITITLE(I),I=7,12)                                      00370
   27 FORMAT(/,69HO  XSCALE     YSCALE      IXUP    IYRIGH     IWIDE       63
     1IHIGH      IGRID,/)
      PRINT 27                                                          00640
      READ 204,XSCALE,YSCALE,IUP,IRITE,IW,IHI,IGR                          65
      PRINT 208,XSCALE,YSCALE,IUP,IRITE,IW,IHI,IGR
   28 FORMAT  (/,37HOORDER OF THE CHARACTERISTIC EQUATION,/)
      PRINT 28                                                          00390
      READ 203,NO                                                       00400
      PRINT 203,NO                                                      00410
      N=NO+1                                                            00420
  205 FORMAT(8E10.5)                                                    00430
  207 FORMAT (8E12.5)                                                   00440
   22 FORMAT  (/,54HOCONSTANT PART OF THE COEFFICIENTS IN DESCENDING ORD   45
     1ER,/)
      PRINT 22                                                          00470
      READ 205,(A(K),K=1,N)                                             00480
      PRINT 207,(A(K),K=1,N)                                            00490
   23 FORMAT  (/,54HOVARIABLE PART OF THE COEFFICIENTS IN DESCENDING ORD   50
     1ER,/)
      PRINT 23                                                          00520
      READ 205,(B(K),K=1,N)                                             00530
      PRINT 207,(B(K),K=1,N)                                            00540
   25 FORMAT  (/,30HOINITIAL VALUE OF THE VARIABLE,/)                      55
      PRINT 25                                                          00560
      READ 204,VAR                                                      00570
      PRINT 2044,VAR                                                    00570
 2044 FORMAT(F10.4)
   26 FORMAT  (/,32HONUMBER OF DECADES TO BE SPANNED,/)                    59
      PRINT 26                                                          00600
      READ 203,ND                                                       00610
      PRINT 203,ND                                                      00620
  201 FORMAT(21HOTHE SYSTEM POLES ARE)                                     63
      PRINT 201                                                         00690
```

```
      M=N                                                                00700
      DO 67 K=1,N                                                        00710
      AP(K)=A(M)                                                         00720
   67 M=M-1
      DO 432 K=1,30                                                      00740
      ROOTI(K)=0.                                                        00750
  432 ROOTR(K)=0.                                                        00760
      NX=NO                                                              00770
      DO 15 K=1,31                                                       00780
   15 X(K)=0.0                                                           00790
      CALL POLRT  (AP,X,NX,ROOTR,ROOTI,IER)
      PRINT 69                                                           00810
      PRINT 68,(ROOTR(K),K=1,NO)                                         00820
      PRINT 70                                                           00830
      PRINT 68,(ROOTI(K),K=1,NO)                                         00840
      CALL DRAW(NO,ROOTR,ROOTI,MOD,1,LABEL,ITITLE,XSCALE,YSCALE,IUP,IRIT
     1E,2,2,IW,IHI,IGR,LAST)
      MOD=2
  202 FORMAT   (21H THE SYSTEM ZEROS ARE,/)                              00880
      K=1                                                                00890
    3 IF(B(K)) 1,2,1                                                     00900
    2 K=K+1                                                              00910
      GO TO 3                                                            00920
    1 NORD=N-K                                                           00930
      IF(NORD-1) 6,4,5                                                   00940
    4 ZERO=-B(K+1)/B(K)                                                  00950
    7 FORMAT   (17H0THE SYSTEM ZERO=,E15.5,/)                               96
      PRINT 7,ZERO                                                       00970
      ROOTM(1)=ZERO                                                      00980
      ROOTM(2)=16.*XSCALE                                                00990
      ROOTJ(1)=0.                                                        01000
      ROOTJ(2)=0.                                                        01010
      NORD=2                                                             01020
      GO TO 11                                                           01030
    6 PRINT 9                                                            01040
    9 FORMAT   (26H0ALL ZEROS ARE AT INFINITY)                             105
      GO TO 8                                                            01060
    5 NN=NORD+1                                                          01070
      DO 10 L=1,NN                                                       01080
      R(L)=B(K)                                                          01090
   10 K=K+1                                                              01100
      PRINT 202                                                          01110
      M=NN                                                               01120
      DO 46 K=1,NN                                                       01130
      AZ(K)=R(M)                                                         01140
   46 M=M-1
      DO 433 K=1,30                                                      01160
      ROOTM(K)=0.                                                        01170
```

```
433 ROOTJ(K)=0.                                                         01180
    NORX=NORD                                                           01190
    DO 16 K=1,31                                                        01200
 16 X(K)=0.0                                                            01210
    CALL POLRT (AZ,X,NORX,ROOTM,ROOTJ,IER)
    PRINT 69                                                            01230
    PRINT 68,(ROOTM(K),K=1,NORD)                                        01240
    PRINT 70                                                            01250
    PRINT 68,(ROOTJ(K),K=1,NORD)                                        01260
 11 CALL DRAW(NORD,ROOTM,ROOTJ,MOD,3,LABEL,ITITLE,XSCALE,YSCALE,
   1IUP,IRITE,2,2,IW,IHI,IGR,LAST)
  8 CONTINUE                                                            01290
    GO TO(31,32,33,34,35,36,37,38,39,40),ND                             01300
 31 G=1.0076                                                            01310
    GO TO 41                                                            01320
 32 G=1.016                                                             01330
    GO TO 41                                                            01340
 33 G=1.0245                                                            01350
    GO TO 41                                                            01360
 34 G=1.0312                                                            01370
    GO TO 41                                                            01380
 35 G=1.0394                                                            01390
    GO TO 41                                                            01400
 36 G=1.0483                                                            01410
    GO TO 41                                                            01420
 37 G=1.0568                                                            01430
    GO TO 41                                                            01440
 38 G=1.0633                                                            01450
    GO TO 41                                                            01460
 39 G=1.071                                                             01470
    GO TO 41                                                            01480
 40 G=1.078                                                             01490
 41 PRINT 30                                                            01500
 30 FORMAT(1H1,///,62HOROOTS FOR THE SPECIFIED VALUES OF THE VARIABLE   01510
   1ARE AS FOLLOWS,/)
    PRINT 42                                                            01530
 42 FORMAT(5X,4H VAR,5X,10HREAL  PART,8X,10HIMAG  PART,13X,10HREAL  PA
   1RT,8X,10HIMAG  PART,13X,10HREAL  PART,8X,10HIMAG  PART,///)
    DO 101 J=1,30                                                       01560
    PRINT 60,VAR
    PRINT 50,((ROOTR(I),ROOTI(I)),I=1,NO)
 60 FORMAT(1PE18.8)
 50 FORMAT(10X,1P2E18.8,5X,1P2E18.8,5X,1P2E18.8)
    DO 100 KAY=1,10
    MR=N
    DO 300 L=1,N
    AP(L)=A(MR)+B(MR)*VAR
300 MR=MR-1
```

```
      DO 434 K=1,30
      ROOTI(K)=0.0
  434 ROOTR(K)=0.0
      NX=NO
      DO 83 K=1,31
   83 X(K)=0.0
      CALL POLRT(AP,X,NX,ROOTR,ROOTI,IER)
      DO 71 JJ=1,NO
      IF(ABS(ROOTI(JJ))-5.E-04) 61,61,71
   61 ROOTI(JJ)=0.0
   71 CONTINUE
  100 VAR=VAR*G
      IF(J-30) 101,98,98
   98 MOD=3
  101 CALL DRAW(NO,ROOTR,ROOTI,MOD,2,LABEL,ITITLE,XSCALE,YSCALE,
     1IUP,IRITE,2,2,IW,IHI,IGR,LAST)
      GO TO 206
      END
//GO.SYSIN DD *
```

```
//ASTOF JOB (1025,0732,EA32),'ASTORQUIZA',TIME=20
// EXEC DSL,REGION.C=150K
//DSL.FT06F001 DD SPACE=(TRK,(5,1))
//DSL.INPUT DD *




*      COMPUTER PROGRAM III



*      OPEN LOOP SYSTEM RESPONSE


INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=5


*      HYDRODYNAMIC COEFFICIENTS


PARAM MXUD=-0.0085,XU=-0.0012
PARAM MYR=-0.0051,YRD=-0.0027
PARAM YV=-0.01243,MYVD=-0.015
PARAM NVD=-0.000197,NV=-0.00351
PARAM NR=-0.00227,IZNRD=-0.00078
PARAM YDEL=0.0027,NDEL=-0.00126
PARAM XN=.00005


*      INITIAL CONDITIONS


INCON Y10=-.1,Y20=.1
INCON X10=1.,X20=0.
INCON U10=1.,U20=1.
PARAM DD1=0.,DD2=0.
PARAM DN1=0.,DN2=0.
PARAM YY1=0.,YY2=0.,YN1=0.,YN2=0.
INITIAL


*      CALCULATION OF THE COEFFICIENTS


        A11=-MYVD
        B11=-YV
        A21=-YRD
        B21=-MYR
        C11=0.
        C12=0.
        C21=0.
        A12=-NVD
        B12=-NV
        A22=-IZNRD
        B22=-NR
        C22=0.
        A33=-MXUD
        B33=-XU
        P=A33/B33
        KC1=XN
        KK1=KC1/B33
        KA=YDEL
        KB=NDEL
        D=A11*A22-A12*A21
```

```
*     INITIAL SEPARATION

       DYO=Y20-Y10
       DXO=X20-X10

       CALL SLOPES(DXO,DYO,YY1,YY2,YN1,YN2)



DERIVATIVE

*     SIMULATION

       DX=X2-X1
       DY=Y2-Y1
       YDOT1=CDOT1*SIN(B1)+ADOT1*COS(B1)
       YDOT2=CDOT2*SIN(B2)+ADOT2*COS(B2)
       XDOT1=CDOT1*COS(B1)-ADOT1*SIN(B1)
       XDOT2=CDOT2*COS(B2)-ADOT2*SIN(B2)
       ADD1=(A22*I11-A21*I21)/D
       ADD2=(A22*I12-A21*I22)/D
       BDD1=(A11*I21-A12*I11)/D
       BDD2=(A11*I22-A12*I12)/D
       ADOT1=INTGRL(0.,ADD1)
       ADOT2=INTGRL(0.,ADD2)
       BDOT1=INTGRL(0.,BDD1)
       BDOT2=INTGRL(0.,BDD2)
       CD1=REALPL(0.,P,KK1*DN1)
       CDOT1=U10+CD1
       CD2=REALPL(0.,P,KK1*DN2)
       CDOT2=U20
       A1=INTGRL(0.,ADOT1)
       A2=INTGRL(0.,ADOT2)
       B1=INTGRL(0.,BDOT1)
       B2=INTGRL(0.,BDOT2)
       Y1=INTGRL(Y10,YDOT1)
       Y2=INTGRL(Y20,YDOT2)
       X1=INTGRL(X10,XDOT1)
       X2=INTGRL(X20,XDOT2)
       I11=-B11*ADOT1-C11*A1-B21*BDOT1-C21*B1+IF11
       I12=-B11*ADOT2-C11*A2-B21*BDOT2-C21*B2+IF12
       I21=-B12*ADOT1-C12*A1-B22*BDOT1-C22*B1+IF21
       I22=-B12*ADOT2-C12*A2-B22*BDOT2-C22*B2+IF22
       AF11=REALPL(0.,0.1,KA*DD1)
       AF12=REALPL(0.,0.1,KA*DD2)
       AF21=REALPL(0.,0.1,KB*DD1)
       AF22=REALPL(0.,0.1,KB*DD2)
       IF11=AF11+YY1
       IF12=AF12+YY2
       IF21=AF21+YN1
       IF22=AF22+YN2
       B1G=57.3*B1
       B2G=57.3*B2

DYNAMIC

*     ACTUAL SEPARATION

       DX=X2-X1
       DY=Y2-Y1

       CALL SLOPES(DX,DY,YY1,YY2,YN1,YN2)
```

```
SAMPLE
PRINT .036,DX,DY,YY1,YY2,YN1,YN2,B1G,B2G,X1,X2
PREPAR .0018,B1G,B2G,Y1,Y2
CONTRL FINTIM=3.2,DELT=.0018,DELS=.0018
       IF(ABS(DY).LE..05)WRITE(6,3)
    3  FORMAT(' ','LATERAL SEPARATION LESS THAN 25 FT')
       CALL DRWG(1,1,TIME,Y1)
       CALL DRWG(1,2,TIME,Y2)
       CALL DRWG(2,1,TIME,B1G)
       CALL DRWG(2,2,TIME,B2G)
TERMINAL
       CALL ENDRW (NPLOT)
END
INCON Y10=-.1,Y20=.1
INCON U10=1.,U20=1.
PARAM X10=0.,X20=0.
END
PARAM Y10=-.05,Y20=.05,X10=0.,X20=0.
END
PARAM U20=1.2,Y10=-.1,Y20=.1,X10=1.,X20=0.
END
PARAM U20=1.2,Y10=-.1,Y20=.1,X10=0.,X20=0.
END
STOP
//C.SYSPRINT DD SPACE=(TRK,(2,2))
//PLOT.STEPLIB  DD  DSN=SYS3.DSLPLOT,UNIT=2321,VOL=SER=CEL00
//PLOT.SYSIN DD *
```

```
      SUBROUTINE SLOPES(DX,DY,YY1,YY2,YN1,YN2)

*     TABLE LOOK-UP AND INTERPOLATION

      DIMENSION Z(23,16),W(23,16),X(23),Y(16)
      X(1)=-1.1
      X(2)=-1.
      X(3)=-.9
      X(4)=-.8
      X(5)=-.7
      X(6)=-.6
      X(7)=-.5
      X(8)=-.4
      X(9)=-.3
      X(10)=-.2
      X(11)=-.1
      X(12)=0.
      X(13)=.1
      X(14)=.2
      X(15)=.3
      X(16)=.4
      X(17)=.5
      X(18)=.6
      X(19)=.7
      X(20)=.8
      X(21)=.9
      X(22)=1.
      X(23)=1.1
      Y(1)=0.10
      Y(2)=0.12
      Y(3)=0.14
      Y(4)=0.16
      Y(5)=0.18
      Y(6)=0.2
      Y(7)=0.22
      Y(8)=0.24
      Y(9)=0.26
      Y(10)=0.28
      Y(11)=0.30
      Y(12)=0.32
      Y(13)=0.34
      Y(14)=0.36
      Y(15)=0.38
      Y(16)=0.40
      Z(1,1)=0.
      Z(1,2)=0.
      Z(1,3)=0.
      Z(1,4)=0.
      Z(1,5)=0.
      Z(1,6)=0.
      Z(1,7)=0.
      Z(1,8)=0.
      Z(1,9)=0.
      Z(1,10)=0.
      Z(1,11)=0.
      Z(1,12)=0.
      Z(1,13)=0.
      Z(1,14)=0.
      Z(1,15)=0.
      Z(1,16)=0.
      Z(2,1)=-24.
      Z(2,2)=-21.
      Z(2,3)=-19.
      Z(2,4)=-21.
      Z(2,5)=-15.
      Z(2,6)=-14.
      Z(2,7)=-12.
      Z(2,8)=-10.
      Z(2,9)=-8.
      Z(2,10)=-6.
```

```
Z(2,11)=-4.
Z(2,12)=-2.
Z(2,13)=0.
Z(2,14)=0.
Z(2,15)=0.
Z(2,16)=0.
Z(3,1)=-36.
Z(3,2)=-29.
Z(3,3)=-26.
Z(3,4)=-22.
Z(3,5)=-18.
Z(3,6)=-16.
Z(3,7)=-14.
Z(3,8)=-12.
Z(3,9)=-10.
Z(3,10)=-8.
Z(3,11)=-6.
Z(3,12)=-4.
Z(3,13)=-2.
Z(3,14)=0.
Z(3,15)=0.
Z(3,16)=0.
Z(4,1)=-39.
Z(4,2)=-33.
Z(4,3)=-28.
Z(4,4)=-23.
Z(4,5)=-18.
Z(4,6)=-16.
Z(4,7)=-14.
Z(4,8)=-12.
Z(4,9)=-10.
Z(4,10)=-8.
Z(4,11)=-6.
Z(4,12)=-4.
Z(4,13)=-2.
Z(4,14)=0.
Z(4,15)=0.
Z(4,16)=0.
Z(5,1)=-38.
Z(5,2)=-31.
Z(5,3)=-26.
Z(5,4)=-21.
Z(5,5)=-18.
Z(5,6)=-14.
Z(5,7)=-12.
Z(5,8)=-10.
Z(5,9)=-8.
Z(5,10)=-6.
Z(5,11)=-4.
Z(5,12)=-2.
Z(5,13)=0.
Z(5,14)=0.
Z(5,15)=0.
Z(5,16)=0.
Z(6,1)=-28.
Z(6,2)=-24.
Z(6,3)=-20.
Z(6,4)=-15.
Z(6,5)=-12.
Z(6,6)=-10.
Z(6,7)=-8.
Z(6,8)=-6.
Z(6,9)=-4.
Z(6,10)=-2.
Z(6,11)=0.
Z(6,12)=0.
Z(6,13)=0.
Z(6,14)=0.
Z(6,15)=0.
Z(6,16)=0.
Z(7,1)=-15.
Z(7,2)=-11.
```

```
Z(7,3)=-9.
Z(7,4)=-6.
Z(7,5)=-4.
Z(7,6)=-2.
Z(7,7)=0.
Z(7,8)=0.
Z(7,9)=0.
Z(7,10)=0.
Z(7,11)=0.
Z(7,12)=0.
Z(7,13)=0.
Z(7,14)=0.
Z(7,15)=0.
Z(7,16)=0.
Z(8,1)=4.
Z(8,2)=6.
Z(8,3)=7.
Z(8,4)=8.
Z(8,5)=9.
Z(8,6)=10.
Z(8,7)=7.
Z(8,8)=6.
Z(8,9)=4.
Z(8,10)=2.
Z(8,11)=0.
Z(8,12)=0.
Z(8,13)=0.
Z(8,14)=0.
Z(8,15)=0.
Z(8,16)=0.
Z(9,1)=27.
Z(9,2)=25.
Z(9,3)=23.
Z(9,4)=22.
Z(9,5)=21.
Z(9,6)=20.
Z(9,7)=18.
Z(9,8)=16.
Z(9,9)=14.
Z(9,10)=12.
Z(9,11)=10.
Z(9,12)=8.
Z(9,13)=6.
Z(9,14)=4.
Z(9,15)=2.
Z(9,16)=0.
Z(10,1)=52.
Z(10,2)=47.
Z(10,3)=43.
Z(10,4)=39.
Z(10,5)=36.
Z(10,6)=34.
Z(10,7)=32.
Z(10,8)=30.
Z(10,9)=28.
Z(10,10)=26.
Z(10,11)=24.
Z(10,12)=22.
Z(10,13)=20.
Z(10,14)=18.
Z(10,15)=16.
Z(10,16)=14.
Z(11,1)=72.
Z(11,2)=64.
Z(11,3)=58.
Z(11,4)=52.
Z(11,5)=46.
Z(11,6)=43.
Z(11,7)=40.
Z(11,8)=37.
Z(11,9)=34.
Z(11,10)=31.
```

```
Z(11,11)=28.
Z(11,12)=25.
Z(11,13)=22.
Z(11,14)=19.
Z(11,15)=17.
Z(11,16)=15.
Z(12,1)=86.
Z(12,2)=75.
Z(12,3)=67.
Z(12,4)=60.
Z(12,5)=53.
Z(12,6)=48.
Z(12,7)=43.
Z(12,8)=39.
Z(12,9)=35.
Z(12,10)=32.
Z(12,11)=29.
Z(12,12)=26.
Z(12,13)=23.
Z(12,14)=20.
Z(12,15)=18.
Z(12,16)=16.
Z(13,1)=89.
Z(13,2)=78.
Z(13,3)=69.
Z(13,4)=60.
Z(13,5)=53.
Z(13,6)=48.
Z(13,7)=43.
Z(13,8)=38.
Z(13,9)=34.
Z(13,10)=31.
Z(13,11)=28.
Z(13,12)=25.
Z(13,13)=22.
Z(13,14)=19.
Z(13,15)=17.
Z(13,16)=15.
Z(14,1)=80.
Z(14,2)=70.
Z(14,3)=63.
Z(14,4)=55.
Z(14,5)=50.
Z(14,6)=45.
Z(14,7)=40.
Z(14,8)=36.
Z(14,9)=33.
Z(14,10)=30.
Z(14,11)=29.
Z(14,12)=26.
Z(14,13)=23.
Z(14,14)=20.
Z(14,15)=18.
Z(14,16)=16.
Z(15,1)=64.
Z(15,2)=56.
Z(15,3)=51.
Z(15,4)=46.
Z(15,5)=41.
Z(15,6)=38.
Z(15,7)=36.
Z(15,8)=34.
Z(15,9)=32.
Z(15,10)=30.
Z(15,11)=28.
Z(15,12)=26.
Z(15,13)=24.
Z(15,14)=22.
Z(15,15)=20.
Z(15,16)=18.
Z(16,1)=45.
Z(16,2)=41.
```

```
Z(16,3)=38.
Z(16,4)=35.
Z(16,5)=33.
Z(16,6)=30.
Z(16,7)=28.
Z(16,8)=26.
Z(16,9)=24.
Z(16,10)=22.
Z(16,11)=20.
Z(16,12)=18.
Z(16,13)=16.
Z(16,14)=14.
Z(16,15)=12.
Z(16,16)=10.
Z(17,1)=30.
Z(17,2)=28.
Z(17,3)=26.
Z(17,4)=24.
Z(17,5)=23.
Z(17,6)=22.
Z(17,7)=20.
Z(17,8)=18.
Z(17,9)=16.
Z(17,10)=14.
Z(17,11)=10.
Z(17,12)=10.
Z(17,13)=8.
Z(17,14)=6.
Z(17,15)=4.
Z(17,16)=2.
Z(18,1)=17.
Z(18,2)=17.
Z(18,3)=16.
Z(18,4)=16.
Z(18,5)=16.
Z(18,6)=16.
Z(18,7)=15.
Z(18,8)=15.
Z(18,9)=15.
Z(18,10)=14.
Z(18,11)=14.
Z(18,12)=14.
Z(18,13)=12.
Z(18,14)=12.
Z(18,15)=12.
Z(18,16)=10.
Z(19,1)=6.
Z(19,2)=7.
Z(19,3)=7.
Z(19,4)=8.
Z(19,5)=8.
Z(19,6)=9.
Z(19,7)=9.
Z(19,8)=8.
Z(19,9)=8.
Z(19,10)=7.
Z(19,11)=6.
Z(19,12)=4.
Z(19,13)=5.
Z(19,14)=3.
Z(19,15)=2.
Z(19,16)=0.
Z(20,1)=-5.
Z(20,2)=-4.
Z(20,3)=-3.
Z(20,4)=-1.
Z(20,5)=0.
Z(20,6)=0.
Z(20,7)=0.
Z(20,8)=0.
Z(20,9)=0.
Z(20,10)=0.
```

```
Z(20,11)=0.
Z(20,12)=0.
Z(20,13)=0.
Z(20,14)=0.
Z(20,15)=0.
Z(20,16)=0.
Z(21,1)=-5.
Z(21,2)=-4.
Z(21,3)=-3.
Z(21,4)=-1.
Z(21,5)=0.
Z(21,6)=0.
Z(21,7)=0.
Z(21,8)=0.
Z(21,9)=0.
Z(21,10)=0.
Z(21,11)=0.
Z(21,12)=0.
Z(21,13)=0.
Z(21,14)=0.
Z(21,15)=0.
Z(21,16)=0.
Z(22,1)=-6.
Z(22,2)=-5.
Z(22,3)=-5.
Z(22,4)=-4.
Z(22,5)=-4.
Z(22,6)=-4.
Z(22,7)=-3.
Z(22,8)=-3.
Z(22,9)=-2.
Z(22,10)=-2.
Z(22,11)=-1.
Z(22,12)=-1.
Z(22,13)=-1.
Z(22,14)=0.
Z(22,15)=0.
Z(22,16)=0.
Z(23,1)=0.
Z(23,2)=0.
Z(23,3)=0.
Z(23,4)=0.
Z(23,5)=0.
Z(23,6)=0.
Z(23,7)=0.
Z(23,8)=0.
Z(23,9)=0.
Z(23,10)=0.
Z(23,11)=0.
Z(23,12)=0.
Z(23,13)=0.
Z(23,14)=0.
Z(23,15)=0.
Z(23,16)=0.
W(1,1)=0.
W(1,2)=0.
W(1,3)=0.
W(1,4)=0.
W(1,5)=0.
W(1,6)=0.
W(1,7)=0.
W(1,8)=0.
W(1,9)=0.
W(1,10)=0.
W(1,11)=0.
W(1,12)=0.
W(1,13)=0.
W(1,14)=0.
W(1,15)=0.
W(1,16)=0.
W(2,1)=11.
W(2,2)=10.
```

```
W(2,3)=10.
W(2,4)=9.
W(2,5)=9.
W(2,6)=8.
W(2,7)=6.
W(2,8)=4.
W(2,9)=2.
W(2,10)=0.
W(2,11)=0.
W(2,12)=0.
W(2,13)=0.
W(2,14)=0.
W(2,15)=0.
W(2,16)=0.
W(3,1)=14.
W(3,2)=14.
W(3,3)=11.
W(3,4)=10.
W(3,5)=9.
W(3,6)=8.
W(3,7)=7.
W(3,8)=6.
W(3,9)=5.
W(3,10)=4.
W(3,11)=3.
W(3,12)=2.
W(3,13)=1.
W(3,14)=0.
W(3,15)=0.
W(3,16)=0.
W(4,1)=14.
W(4,2)=12.
W(4,3)=10.
W(4,4)=8.
W(4,5)=7.
W(4,6)=6.
W(4,7)=5.
W(4,8)=4.
W(4,9)=3.
W(4,10)=2.
W(4,11)=1.
W(4,12)=0.
W(4,13)=0.
W(4,14)=0.
W(4,15)=0.
W(4,16)=0.
W(5,1)=10.
W(5,2)=6.
W(5,3)=5.
W(5,4)=4.
W(5,5)=3.
W(5,6)=2.
W(5,7)=1.
W(5,8)=0.
W(5,9)=0.
W(5,10)=0.
W(5,11)=0.
W(5,12)=0.
W(5,13)=0.
W(5,14)=0.
W(5,15)=0.
W(5,16)=0.
W(6,1)=-5.
W(6,2)=-5.
W(6,3)=-4.
W(6,4)=-4.
W(6,5)=-4.
W(6,6)=-4.
W(6,7)=-3.
W(6,8)=-3.
W(6,9)=-3.
W(6,10)=-2.
```

```
W(6,11)=-2.
W(6,12)=-1.
W(6,13)=-1.
W(6,14)=0.
W(6,15)=0.
W(6,16)=0.
W(7,1)=-18.
W(7,2)=-16.
W(7,3)=-14.
W(7,4)=-12.
W(7,5)=-10.
W(7,6)=-9.
W(7,7)=-8.
W(7,8)=-7.
W(7,9)=-6.
W(7,10)=-5.
W(7,11)=-4.
W(7,12)=-3.
W(7,13)=-2.
W(7,14)=-1.
W(7,15)=0.
W(7,16)=0.
W(8,1)=-30.
W(8,2)=-26.
W(8,3)=-22.
W(8,4)=-19.
W(8,5)=-17.
W(8,6)=-15.
W(8,7)=-13.
W(8,8)=-11.
W(8,9)=-9.
W(8,10)=-7.
W(8,11)=-5.
W(8,12)=-3.
W(8,13)=-1.
W(8,14)=0.
W(8,15)=0.
W(8,16)=0.
W(9,1)=-41.
W(9,2)=-35.
W(9,3)=-30.
W(9,4)=-25.
W(9,5)=-22.
W(9,6)=-18.
W(9,7)=-16.
W(9,8)=-14.
W(9,9)=-12.
W(9,10)=-10.
W(9,11)=-8.
W(9,12)=-6.
W(9,13)=-4.
W(9,14)=-2.
W(9,15)=0.
W(9,16)=0.
W(10,1)=-44.
W(10,2)=-38.
W(10,3)=-33.
W(10,4)=-28.
W(10,5)=-25.
W(10,6)=-20.
W(10,7)=-17.
W(10,8)=-14.
W(10,9)=-11.
W(10,10)=-9.
W(10,11)=-7.
W(10,12)=-5.
W(10,13)=-3.
W(10,14)=-2.
W(10,15)=-1.
W(10,16)=0.
W(11,1)=-43.
W(11,2)=-37.
```

```
W(11,3)=-33.
W(11,4)=-28.
W(11,5)=-25.
W(11,6)=-20.
W(11,7)=-17.
W(11,8)=-14.
W(11,9)=-11.
W(11,10)=-9.
W(11,11)=-7.
W(11,12)=-5.
W(11,13)=-3.
W(11,14)=-2.
W(11,15)=-1.
W(11,16)=0.
W(12,1)=-37.
W(12,2)=-33.
W(12,3)=-30.
W(12,4)=-26.
W(12,5)=-23.
W(12,6)=-20.
W(12,7)=-17.
W(12,8)=-14.
W(12,9)=-11.
W(12,10)=-9.
W(12,11)=-7.
W(12,12)=-5.
W(12,13)=-3.
W(12,14)=-2.
W(12,15)=-1.
W(12,16)=0.
W(13,1)=-29.
W(13,2)=-26.
W(13,3)=-25.
W(13,4)=-22.
W(13,5)=-19.
W(13,6)=-17.
W(13,7)=-15.
W(13,8)=-13.
W(13,9)=-11.
W(13,10)=-9.
W(13,11)=-7.
W(13,12)=-5.
W(13,13)=-3.
W(13,14)=-2.
W(13,15)=-1.
W(13,16)=0.
W(14,1)=-18.
W(14,2)=-16.
W(14,3)=-16.
W(14,4)=-13.
W(14,5)=-12.
W(14,6)=-10.
W(14,7)=-9.
W(14,8)=-8.
W(14,9)=-7.
W(14,10)=-6.
W(14,11)=-5.
W(14,12)=-4.
W(14,13)=-3.
W(14,14)=-2.
W(14,15)=-1.
W(14,16)=0.
W(15,1)=-5.
W(15,2)=-4.
W(15,3)=-4.
W(15,4)=-4.
W(15,5)=-4.
W(15,6)=-4.
W(15,7)=-3.
W(15,8)=-3.
W(15,9)=-3.
W(15,10)=-3.
```

```
W(15,11)=-2.
W(15,12)=-2.
W(15,13)=-1.
W(15,14)=-1.
W(15,15)=0.
W(15,16)=0.
W(16,1)=-4.
W(16,2)=-4.
W(16,3)=-3.
W(16,4)=-3.
W(16,5)=-2.
W(16,6)=-2.
W(16,7)=-2.
W(16,8)=-1.
W(16,9)=-1.
W(16,10)=-1.
W(16,11)=0.
W(16,12)=0.
W(16,13)=0.
W(16,14)=0.
W(16,15)=0.
W(16,16)=0.
W(17,1)=11.
W(17,2)=10.
W(17,3)=9.
W(17,4)=8.
W(17,5)=7.
W(17,6)=6.
W(17,7)=5.
W(17,8)=4.
W(17,9)=3.
W(17,10)=2.
W(17,11)=1.
W(17,12)=0.
W(17,13)=0.
W(17,14)=0.
W(17,15)=0.
W(17,16)=0.
W(18,1)=14.
W(18,2)=12.
W(18,3)=11.
W(18,4)=10.
W(18,5)=9.
W(18,6)=8.
W(18,7)=6.
W(18,8)=4.
W(18,9)=2.
W(18,10)=0.
W(18,11)=0.
W(18,12)=0.
W(18,13)=0.
W(18,14)=0.
W(18,15)=0.
W(18,16)=0.
W(19,1)=13.
W(19,2)=12.
W(19,3)=10.
W(19,4)=10.
W(19,5)=9.
W(19,6)=8.
W(19,7)=6.
W(19,8)=4.
W(19,9)=2.
W(19,10)=0.
W(19,11)=0.
W(19,12)=0.
W(19,13)=0.
W(19,14)=0.
W(19,15)=0.
W(19,16)=0.
W(20,1)=12.
W(20,2)=10.
```

```
W(20,3)=9.
W(20,4)=8.
W(20,5)=7.
W(20,6)=7.
W(20,7)=6.
W(20,8)=5.
W(20,9)=4.
W(20,10)=3.
W(20,11)=2.
W(20,12)=1.
W(20,13)=0.
W(20,14)=0.
W(20,15)=0.
W(20,16)=0.
W(21,1)=10.
W(21,2)=9.
W(21,3)=7.
W(21,4)=6.
W(21,5)=5.
W(21,6)=4.
W(21,7)=3.
W(21,8)=2.
W(21,9)=1.
W(21,10)=0.
W(21,11)=0.
W(21,12)=0.
W(21,13)=0.
W(21,14)=0.
W(21,15)=0.
W(21,16)=0.
W(22,1)=5.
W(22,2)=4.
W(22,3)=3.
W(22,4)=3.
W(22,5)=2.
W(22,6)=2.
W(22,7)=1.
W(22,8)=1.
W(22,9)=0.
W(22,10)=0.
W(22,11)=0.
W(22,12)=0.
W(22,13)=0.
W(22,14)=0.
W(22,15)=0.
W(22,16)=0.
W(23,1)=0.
W(23,2)=0.
W(23,3)=0.
W(23,4)=0.
W(23,5)=0.
W(23,6)=0.
W(23,7)=0.
W(23,8)=0.
W(23,9)=0.
W(23,10)=0.
W(23,11)=0.
W(23,12)=0.
W(23,13)=0.
W(23,14)=0.
W(23,15)=0.
W(23,16)=0.
IF(ABS(DY).GE..4734) GO TO 1
I=IFIX((DX+1.1002)/.1)+1.
J=IFIX((DY-.1)/.02)+1.
IF(J.LT.1)J=1
IF(I.LT.1)I=1
IF(I.GT.23)I=23
IF(J.GT.16)J=16
K=24-I
DELX=DX-X(I)
DELY=DY-Y(J)
```

```
      DELK=DX-X(K)
      IF((I.EQ.23).OR.(J.EQ.16)) GO TO 2
      DYD1=DELX*(Z(I+1,J)-Z(I,J))+DELY*(Z(I,J+1)-Z(I,J))
      DYD2=DELK*(Z(K+1,J)-Z(K,J))+DELY*(Z(K,J+1)-Z(K,J))
      DYND1=DELX*(W(I+1,J)-W(I,J))+DELY*(W(I,J+1)-W(I,J))
      DYND2=DELK*(W(K+1,J)-W(K,J))+DELY*(W(K,J+1)-W(K,J))
      YY1=(Z(I,J)+DYD1)*1.E-05
      YY2=-(Z(K,J)+DYD2)*1.E-05
      YN1=(W(I,J)+DYND1)*1.E-05
      YN2=-(W(K,J)+DYND2)*1.E-05
      RETURN
1     YY1=0.
      YY2=0.
      YN1=0.
      YN2=0.
      RETURN
2     YY1=Z(I,J)*1.E-05
      YY2=-Z(K,J)*1.E-05
      YN1=W(I,J)*1.E-05
      YN2=-W(K,J)*1.E-05
      RETURN
      END
```

```
//ASTOF JOB (1025,0732,EA32),'ASTORQUIZA',TIME=20
// EXEC DSL,REGION.C=150K
//DSL.FT06F001 DD SPACE=(TRK,(5,1))
//DSL.INPUT DD *




*       COMPUTER PROGRAM IV



*       CONTROLLED PLANT RESPONSE


INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=2


*       HYDRODYNAMIC COEFFICIENTS


PARAM MXUD=-0.0085,XU=-0.0012
PARAM MYR=-0.0051,YRD=-0.0027
PARAM YV=-0.01243,MYVD=-0.015
PARAM NVD=-0.000197,NV=-0.00351
PARAM NR=-0.00227,IZNRD=-0.00078
PARAM YDEL=0.0027,NDEL=-0.00126
PARAM XN=.00005
PARAM TOP=-.3490401396


*       INITIAL CONDITIONS


INCON Y10=-.2,Y20=.2
INCON X10=1.,X20=0
INCON U10=1.,U20=1.2
PARAM DN1=0.,DN2=0
PARAM YY1=0.,YY2=0.,YN1=0.,YN2=0.


*       DESIRED FINAL SEPARATION


PARAM DFIN=.24


*       FEEDBACK LOOP GAINS


PARAM K2=2.917,KT2=2.042,K1=2.975,KT1=2.775
PARAM KY2=1.511,KTY2=2.878,KY1=3.008,KTY1=2.783


INITIAL


*       CALCULATION OF THE COEFFICIENTS


          A11=-MYVD
          B11=-YV
          A21=-YRD
          B21=-MYR
          C11=0.
          C12=0.
          C21=0.
```

```
      A12=-NVD
      B12=-NV
      A22=-IZNRD
      B22=-NR
      C22=0.
      A33=-MXUD
      B33=-XU
      P=A33/B33
      KC1=XN
      KK1=KC1/B33
      KA=YDEL
      KB=NDEL
      D=A11*A22-A12*A21

*    INITIAL SEPARATION


      DY0=Y20-Y10
      DX0=X20-X10


      CALL SLOPES(DX0,DY0,YY1,YY2,YN1,YN2)

DERIVATIVE

*    SIMULATION

      DX=X2-X1
      DY=Y2-Y1
      YDOT1=CDOT1*SIN(B1)+ADOT1*COS(B1)
      YDOT2=CDOT2*SIN(B2)+ADOT2*COS(B2)
      XDOT1=CDOT1*COS(B1)-ADOT1*SIN(B1)
      XDOT2=CDOT2*COS(B2)-ADOT2*SIN(B2)
      ADD1=(A22*I11-A21*I21)/D
      ADD2=(A22*I12-A21*I22)/D
      BDD1=(A11*I21-A12*I11)/D
      BDD2=(A11*I22-A12*I12)/D
      ADOT1=INTGRL(0.,ADD1)
      ADOT2=INTGRL(0.,ADD2)
      BDOT1=INTGRL(0.,BDD1)
      BDOT2=INTGRL(0.,BDD2)
      CD1=REALPL(0.,P,KK1*DN1)
      CDOT1=U10+CD1
      CD2=REALPL(0.,P,KK1*DN2)
      CDOT2=NORMA(DX,U20,CDOT1)
      A1=INTGRL(0.,ADOT1)
      A2=INTGRL(0.,ADOT2)
      B1=INTGRL(0.,BDOT1)
      B2=INTGRL(0.,BDOT2)
      Y1=INTGRL(Y10,YDOT1)
      Y2=INTGRL(Y20,YDOT2)
      X1=INTGRL(X10,XDOT1)
      X2=INTGRL(X20,XDOT2)
      I11=-B11*ADOT1-C11*A1-B21*BDOT1-C21*B1+IF11
      I12=-B11*ADOT2-C11*A2-B21*BDOT2-C21*B2+IF12
      I21=-B12*ADOT1-C12*A1-B22*BDOT1-C22*B1+IF21
      I22=-B12*ADOT2-C12*A2-B22*BDOT2-C22*B2+IF22
      AF11=REALPL(0.,0.1,KA*DD1)
      AF12=REALPL(0.,0.1,KA*DD2)
      AF21=REALPL(0.,0.1,KB*DD1)
      AF22=REALPL(0.,0.1,KB*DD2)
      IF11=AF11+YY1
      IF12=AF12+YY2
      IF21=AF21+YN1
      IF22=AF22+YN2
      B1G=57.3*B1
      B2G=57.3*B2
      DYDOT=YDOT2-YDOT1
```

```
*      DDC=COURSE CONTROL ACTION

*      DDD=DISTANCE CONTROL ACTION

       DDC1=K1*B1+KT1*BDOT1
       DDD1=KY1*Y1+KTY1*YDOT1
       DDC2=K2*B2+KT2*BDOT2
       DDD2=KY2*(DY-DFIN)+KTY2*DYDOT
       DD1=DIDO(DDD1,DDC1,TOP)
       DD2=CARLO(DDC2,DDD2,TOP)
       DD2G=DD2*57.3
       DD1G=DD1*57.3

DYNAMIC

*     ACTUAL SEPARATION

       DX=X2-X1
       DY=Y2-Y1

       CALL SLOPES(DX,DY,YY1,YY2,YN1,YN2)
SAMPLE
CONTRL FINTIM=24.,DELT=.013,DELS=.013
PREPAR .013,B1G,B2G,Y1,Y2,DD1G,DD2G
PRINT .13,DX,DY,YY1,YY2,YN1,YN2,B1G,B2G,X1,X2
       IF(ABS(DY).LE..05)WRITE(6,3)
   3   FORMAT(' ','LATERAL SEPARATION LESS THAN 25 FT')
       CALL DRWG(1,1,TIME,Y1)
       CALL DRWG(1,2,TIME,Y2)
       CALL DRWG(2,1,TIME,B1G)
       CALL DRWG(2,2,TIME,B2G)
       CALL DRWG(3,1,TIME,DD1G)
       CALL DRWG(3,2,TIME,DD2G)
TERMINAL
       CALL ENDRW (NPLOT)
END
PARAM KY2=1.533,KTY2=2.608,KY1=3.32,KTY1=2.555
PARAM K2=2.853,KT2=2.141,K1=3.069,KT1=3.08
INCON Y10=-.18,Y20=.18
PARAM DFIN=.3
END
STOP

FORTRAN

      FUNCTION NORMA(DX,U20,CDOT1)

      CDOT2=U20
      IF(ABS(DX).LE..005) CDOT2=CDOT1
      NORMA=CDOT2
      RETURN
      END

      FUNCTION DIDO(DDC1,DDD1,TOP)

      DD1=DDC1+DDD1
      IF(DD1.LT.TOP) DD1=TOP
      DIDO=DD1
      RETURN
      END
```

```
      FUNCTION CARLO(DDC2,DDD2,TOP)

      DD2=DDC2+DDD2
      IF(DD2.LT.TOP) DD2=TOP
      CARLO=DD2
      RETURN
      END


//C.SYSPRINT DD SPACE=(TRK,(2,2))
//PLOT.STEPLIB  DD  DSN=SYS3.DSLPLOT,UNIT=2321,VOL=SER=CEL00
//PLOT.SYSIN DD *
```

```
//ASTOF JOB (1025,0732,EA32),'ASTORQUIZA',TIME=20
// EXEC DSL,REGION.C=150K
//DSL.FT06F001 DD SPACE=(TRK,(5,1))
//DSL.INPUT DD *




*      COMPUTER PROGRAM V



*      CONTROLLED PLANT RESPONSE


INTEG TRAPZ
INTGER NPLOT
CONST NPLOT=4


*      HYDRODYNAMIC COEFFICIENTS


PARAM MXUD=-0.0085,XU=-0.0012
PARAM MYR=-0.0051,YRD=-0.0027
PARAM YV=-0.01243,MYVD=-0.015
PARAM NVD=-0.000197,NV=-0.00351
PARAM NR=-0.00227,IZNRD=-0.00078
PARAM YDEL=0.0027,NDEL=-0.00126
PARAM XN=.00005
PARAM TOP=-.3490401396


*      INITIAL CONDITIONS


INCON Y10=-.18,Y20=.18
INCON X10=1.,X20=0
INCON U10=1.,U20=1.2
PARAM DN1=0.,DN2=0
PARAM YY1=0.,YY2=0.,YN1=0.,YN2=0.


*      DESIRED FINAL SEPARATION


PARAM DFIN=.3


*      QUASI-OPTIMAL FEEDBACK LOOP GAINS


PARAM KY2=1.533,KTY2=2.608,KY1=3.32,KTY1=2.555
PARAM K2=2.853,KT2=3.2115,K1=3.069,KT1=4.62


INITIAL


*      CALCULATION OF THE COEFFICIENTS


         A11=-MYVD
         B11=-YV
         A21=-YRD
         B21=-MYR
         C11=0.
         C12=0.
         C21=0.
```

```
      A12=-NVD
      B12=-NV
      A22=-IZNRD
      B22=-NR
      C22=0.
      A33=-MXUD
      B33=-XU
      P=A33/B33
      KC1=XN
      KK1=KC1/B33
      KA=YDEL
      KB=NDEL
      D=A11*A22-A12*A21

*    INITIAL SEPARATION

      DY0=Y20-Y10
      DX0=X20-X10

      CALL SLOPES(DX0,DY0,YY1,YY2,YN1,YN2)

DERIVATIVE

*    SIMULATION

      DX=X2-X1
      DY=Y2-Y1
      YDOT1=CDOT1*SIN(B1)+ADOT1*COS(B1)
      YDOT2=CDOT2*SIN(B2)+ADOT2*COS(B2)
      XDOT1=CDOT1*COS(B1)-ADOT1*SIN(B1)
      XDOT2=CDOT2*COS(B2)-ADOT2*SIN(B2)
      ADD1=(A22*I11-A21*I21)/D
      ADD2=(A22*I12-A21*I22)/D
      BDD1=(A11*I21-A12*I11)/D
      BDD2=(A11*I22-A12*I12)/D
      ADOT1=INTGRL(0.,ADD1)
      ADOT2=INTGRL(0.,ADD2)
      BDOT1=INTGRL(0.,BDD1)
      BDOT2=INTGRL(0.,BDD2)
      CD1=REALPL(0.,P,KK1*DN1)
      CDOT1=U10+CD1
      CD2=REALPL(0.,P,KK1*DN2)
      CDOT2=NORMA(DX,U20,CDOT1)
      A1=INTGRL(0.,ADOT1)
      A2=INTGRL(0.,ADOT2)
      B1=INTGRL(0.,BDOT1)
      B2=INTGRL(0.,BDOT2)
      Y1=INTGRL(Y10,YDOT1)
      Y2=INTGRL(Y20,YDOT2)
      X1=INTGRL(X10,XDOT1)
      X2=INTGRL(X20,XDOT2)
      I11=-B11*ADOT1-C11*A1-B21*BDOT1-C21*B1+IF11
      I12=-B11*ADOT2-C11*A2-B21*BDOT2-C21*B2+IF12
      I21=-B12*ADOT1-C12*A1-B22*BDOT1-C22*B1+IF21
      I22=-B12*ADOT2-C12*A2-B22*BDOT2-C22*B2+IF22
      AF11=REALPL(0.,0.1,KA*DD1)
      AF12=REALPL(0.,0.1,KA*DD2)
      AF21=REALPL(0.,0.1,KB*DD1)
      AF22=REALPL(0.,0.1,KB*DD2)
      IF11=AF11+YY1
      IF12=AF12+YY2
      IF21=AF21+YN1
      IF22=AF22+YN2
      B1G=57.3*B1
      B2G=57.3*B2
      DYDOT=YDOT2-YDOT1
```

```
*      DDC=COURSE CONTROL ACTION

*      DDD=DISTANCE CONTROL ACTION

       DDC1=K1*B1+KT1*BDOT1
       DDD1=KY1*Y1+KTY1*YDOT1
       DDC2=K2*B2+KT2*BDOT2
       DDD2=KY2*(DY-DFIN)+KTY2*DYDOT
       DD1=DIDO(DDD1,DDC1,TOP)
       DD2=CARLO(DDC2,DDD2,TOP)
       DD2G=DD2*57.3
       DD1G=DD1*57.3

DYNAMIC

*     ACTUAL SEPARATION

       DX=X2-X1
       DY=Y2-Y1

       CALL SLOPES(DX,DY,YY1,YY2,YN1,YN2)
SAMPLE
CONTRL FINTIM=24.,DELT=.013,DELS=.013
PREPAR .013,B1G,B2G,Y1,Y2,DD1G,DD2G
PRINT .13,DX,DY,YY1,YY2,YN1,YN2,B1G,B2G,X1,X2
       IF(ABS(DY).LE..05)WRITE(6,3)
   3   FORMAT(' ','LATERAL SEPARATION LESS THAN 25 FT')
       CALL DRWG(1,1,TIME,Y1)
       CALL DRWG(1,2,TIME,Y2)
       CALL DRWG(2,1,TIME,B1G)
       CALL DRWG(2,2,TIME,B2G)
TERMINAL
       CALL ENDRW (NPLOT)
END
PARAM K2=2.853,KT2=2.141,K1=3.069,KT1=4.62
PARAM KY2=1.533,KTY2=2.608,KY1=3.32,KTY1=3.8325
END
PARAM K2=2.853,KT2=2.141,K1=3.069,KT1=4.62
PARAM KY2=1.91625,KTY2=2.608,KY1=3.32,KTY1=3.8325
END
PARAM K2=2.853,KT2=2.141,K1=3.52935,KT1=4.62
PARAM KY2=1.91625,KTY2=2.608,KY1=3.32,KTY1=3.8325
END
STOP

FORTRAN

      FUNCTION NORMA(DX,U20,CDOT1)

      CDOT2=U20
      IF(ABS(DX).LE..005) CDOT2=CDOT1
      NORMA=CDOT2
      RETURN
      END
```

```
      FUNCTION DIDO(DDC1,DDD1,TOP)

      DD1=DDC1+DDD1
      IF(DD1.LT.TOP) DD1=TOP
      DIDO=DD1
      RETURN
      END


      FUNCTION CARLO(DDC2,DDD2,TOP)

      DD2=DDC2+DDD2
      IF(DD2.LT.TOP) DD2=TOP
      CARLO=DD2
      RETURN
      END

//C.SYSPRINT DD SPACE=(TRK,(2,2))
//PLOT.STEPLIB  DD  DSN=SYS3.DSLPLOT,UNIT=2321,VOL=SER=CEL00
//PLOT.SYSIN DD *
```

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